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AFIT/GST/ENS/93M-02

A NEW GOODNESS-OF-FIT TEST FOR THE WEIBULL DISTRIBUTION BASED ON SPACINGS

THESIS

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THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science (Operations Research)

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March, 1993

THESIS APPROVAL

STUDENT: Capt Mark C. Coppa

CLASS: GST-93M-02

THESIS TITLE: A New Goodness-of-Fit Test for the Weibull Distribution Based on Spacings

DEFENSE DATE: 8 March 1993

COMMITTEE:	NAME/DEPARTMENT	SIGNATURE
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Preface

The purpose of this thesis is to generate critical values for a new goodness-of-fit test for the Weibull distribution. These critical values are used in an extensive power study to test whether a set of observations follow a Weibull pattern when the shape parameter is known. The Monte Carlo method is used to validate the values obtained and to compare the Z^* test statistic with its prominent competitors.

I would like to express my sincere appreciation to my advisor, Doctor Albert H. Mocre, for his considerable guidance and encouragement throughout this process. Also, the assistance of my reader, Doctor Joseph P. Cain, is greatly appreciated.

I would also like to thank my family and friends for enduring the highs and lows of the past nineteen months.

Finally, I would like to dedicate this thesis effort to the memory of my father, Charles John Coppa, who past away from a prolonged illness this past November.

Mark Charles Coppa

Table of Contents

List of Figures

List of Tables

Table		Page

Table		Page

Table | Page

AFIT/GST/ENS/93M-02

Abstract

The critical values for a new goodness-of-fit test based on spacings are generated for the Weibull distribution when the shape parameter is known. The critical values are used for testing whether a set of observations follow a Weibull distribution when the scale and location parameters are unknown. A Monte Carlo simulation with 10,000 iterations is used to generate the critical values for sample sizes 5(5)35 at shape parameters k = 0.5(0.5)1.5 and for sample sizes 5(5)20 at shape parameters k = 2.0(1.0)4.0.

A Monte Carlo power study of the Z^* test statistic using 5000 iterations is accomplished using nine alternate distributions H_a. The power is good to excellent when the null hypothesis H_0 is from a skewed distribution($k < 2.0$). Power results at shape parameters $k \geq 2.0$ are poor for all sample sizes considered.

A comparison is made, at shape parameter k = 1.0, against the prominent competing goodness-of-fit test statistics. Data is obtained from a prior AFIT thesis by Bush. Results indicate that the Z^* test is more powerful than the competition at sample sizes: 5, 15 and 25 and α levels 0.05 and 0.01.

A relationship between the critical value and the sample size is investigated to allow for greater usage of the test statistic. Satisfactory values of *fit* are attained with a simple linear relationship.

A NEW GOODNESS-OF-FIT TEST FOR THE WEIBULL DISTRIBUTION BASED ON SPACINGS

I. INTRODUCTION

The importance of probabilistic or statistical modeling in the modern world cannot be overrated. The accessibility of high-speed computers has enabled scientists, engineers and analysts to construct and use complex models representing important processes. These models and the associated statistical analyses are of great assistance to decision makers in the fields of medicine, politics and the national defense establishment.

The United States Air Force relies on statistical failure models to describe the failure patterns of various mechanical and electronic components in its weapon systems. The selection of a model to represent a population of entities is a critical and difficult task. Indeed, the choice of an incorrect model can make subsequent analysis useless.

1.1 Background

What is a model? An idealization or abstraction of the real world, a model is an incomplete representation of an entire population. Specifically, a statistical failure model describes the probability of failure for components as a function of time. The model is an analog of reality, made up of factors we assume are representative of the general population. The important concept is whether or not the outputs of the model are reasonably appropriate and valid. One can only speculate about the vast number of wrong decisions made due to the choice of a poor model.

1.2 Goodness-of-fit tests

Fortunately, statistical tests can determine whether sample data correspond to a hypothesized failure model. Through experimentation and observation, time to failure data can be collected. The data set is compared to a theoretical probability distribution. The test to determine if the hypothesized distribution *fits* the data in the sample is called a goodness-of-fit test.

There are several classical goodness-of-fit tests:

- Chi-square goodness of fit test
- Kolmogorov-Smirnov goodness of fit test
- Anderson-Darling goodness of fit test
- Cramer von Mises goodness of fit test

If these tests show a good *fit*, the hypothesized distribution can be used to predict the failure rates of Air Force systems or system components.

1.3 Elements of a Statistical Test

The objective of a statistical test is to test a hypothesis concerning the values of one or more population parameters. We will generally have a theory or research hypothesis that we wish to support or disprove. If sufficient evidence gathered from sample data against our theory is found, we conclude that the theory or null hypothesis is false. The converse of the null hypothesis is called the alternative hypothesis (14:428).

We decide between the null hypothesis, H_0, and the alternative hypothesis, H_a, by evaluating a test statistic based on sample data. If the value of our test statistic falls in a certain range or rejection region, we conclude that H_0 is false. All statistical tests of hypothesis work in the same way and are composed of the same basic elements:

- Null hypothesis, H_0
- Alternative hypothesis, H_a
- Test Statistic
- Rejection region

The functioning parts of the statistical test are the test statistic and the rejection region. The rejection region specifies the value of the test statistic for which the null hypothesis H_0 is rejected.

There are two types of errors that can be made when reaching a decision about the null hypothesis:

- A type I error is made if H_0 is rejected when H_0 is true. The probability of a Type I error is denoted by α.
- A type II error is made if H_0 is accepted when H_a is true. The probability of a Type II error is denoted by β.

Thus, α and β measure the risks associated with making an erroneous decision. As such, they provide a very practical way to measure the goodness of a test(14:429–430).

An additional measure of the goodness of a test is called the power of the test. It is denoted by $1 - \beta$ and is used extensively in this thesis. The power of the test is the probability of rejecting the nuli hypothesis H_0 when it is false.

1.4 Weibull Distribution

There are several failure models and associated goodness-of-fit tests that attempt to satisfy the requirement of modelling component failure patterns. This thesis will only consider the Weibull statistical distribution. The popularity of the Weibull distribution for life data analysis is due primarily to its flexibility. It provides reasonable fit over a wide range of data, and is particularly useful when a skewed or

asymmetric distribution is required(8:452–454). Because of its flexibility and popularity, many inferential techniques are found in the literature. The validity of these techniques depends on the suitability of the Weibull distribution as a model for the data. Thus, valid goodness-of-fit tests for the Weibull distribution are essential.

Goodness-of-fit tests for the Weibull are usually performed on the log transformed data, $x = ln(t)$, where t represents the sample failure times. This transformation makes the mathematical computations more tractable. The transformed distribution is called the log-Weibull or extreme-value distribution. A goodness-of-fit test for the extreme-value distribution is equivalent to a goodness-of-fit test for the Weibull distribution(11:385–387).

1.5 Problem

Current test statistics are based on the log-transformed data rather than on the raw data. My goal is to develop a powerful goodness-of-fit test for the Weibull distribution, using the raw data, that is easy to calculate and implement.

1.6 Objectives

The objectives of this thesis are to:

- Develop a new test statistic for the Weibull distribution
- Generate rejection tables at various alpha levels for this new test statistic
- Conduct a power comparison between the new test statistic and its competition
- Investigate the relationship between the critical values of the test statistic and the size of the sample

II. LITERATURE REVIEW

2.1 History of the Weibull Distribution

The Weibull probability density function or *pdf* was developed in 1939 by a Swedish scientist, Waloddi Weibull. In his study, he examined the distribution of the phenomenon of rupture in solids. In a paper published in 1951, Weibull demonstrated the application of the distribution in an investigation of the yield strength and fatigue of steel, the size distribution of fly ash and the fiber strength of cotton(23:293–297). Although first introduced as a probabilistic characterization for the breaking strength of materials, the Weibull distribution is now widely used for life testing applications in the field of reliability.

2.2 Statistical Properties of the Weibull Distribution

The Weibull distribution is easiest to remember in its cumulative form. The cumulative distribution for a random variable x distributed as the three-parameter Weibull is given by

$$F(x;\theta,\beta,\delta) = 1 - e^{-\left(\frac{x-\delta}{\theta-\delta}\right)^{\beta}} \tag{1}$$

where $\beta > 0$, $\theta > 0$ and $\delta \geq 0$. The parameter β is called the shape parameter; the parameter θ is called the scale parameter or the characteristic life; and the parameter δ is the location parameter or minimum life. When the minimum life equals zero, the three parameter Weibull reduces to the two parameter Weibull whose cumulative distribution function or *cdf* is given by

$$F(x;\theta,\beta) = 1 - e^{-\left(\frac{x}{\theta}\right)^{\beta}} \tag{2}$$

with $x \geq 0$. A simple linear transformation can always convert the three parameter case to the two parameter case. In terms of reliability of system components, the location parameter δ indicates the value of x for which failures may begin to occur. When $\delta = 0$, failures can occur immediately after a component or device is put into operation. A value of $\delta > 0$ indicates a period of time that is failure free. The remaining equations are based on the two parameter case: $\delta = 0$. The probability density function of the two parameter Weibull is given by

$$f(x;\theta,\beta) = \frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} e^{-\left(\frac{x}{\theta}\right)^{\beta}} \tag{3}$$

where

$$f(x) = \frac{dF(x)}{dx} \tag{4}$$

The hazard function is the instantaneous failure rate. The Weibull hazard function can be strictly increasing, strictly decreasing or constant. Since many systems often experience a change in the failure rate, the Weibull can adequately model any of the three situations The hazard function is defined by

$$h(x) = \frac{f(x)}{1 - F(x)} \tag{5}$$

or more specifically for the Weibull two parameter case

$$h(x) = \frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} \tag{6}$$

The value of the shape parameter β determines the failure rate direction:

- When $\beta < 1$, the failure rate will decrease with time. This situation is common during the burn-in period of systems or components.
- When $\beta = 1$, the failure rate will be constant. This describes the useful life period of many systems or components.
- When $\beta > 1$, the failure rate will increase with time. This situation is common during the wearout period.

The mean of the Weibull distribution is

$$\mu = \theta\,\Gamma\left(1 + \frac{1}{\beta}\right) \tag{7}$$

and the variance is given by

$$\sigma^2 = \theta^2\left(\Gamma(1 + \frac{2}{\beta}) - \Gamma^2\,(1 + \frac{1}{\beta})\right) \tag{8}$$

The shape parameter β also determines the shape of the probability density function. As indicated on the accompanying plots, the distribution takes on vastly different forms depending on the value of β. In all plots, the value of the scale parameter $\theta = 1$. For $\beta \leq 1$, the Weibull *pdf* is highly skewed or asymmetric. As β increases above one, the skewness decreases until reaching zero at a value between 3.5 and 4.0.

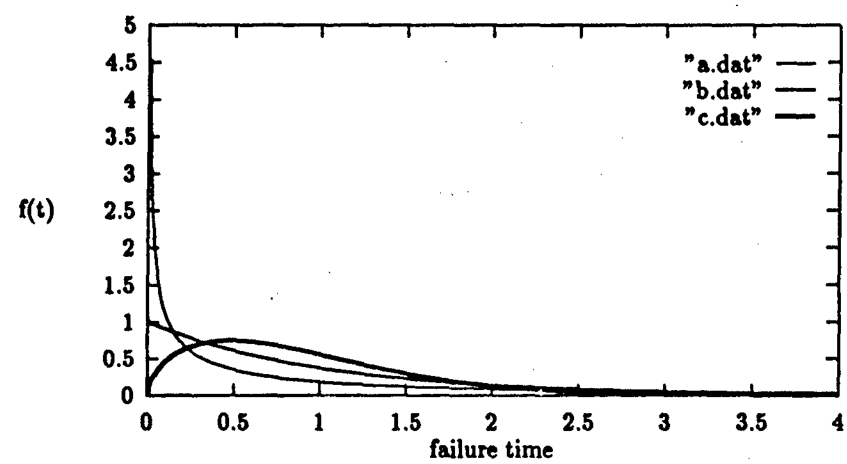

Figure 1. Graph of Weibull distributions: shape parameter k = 0.5, 1.0, 1.5

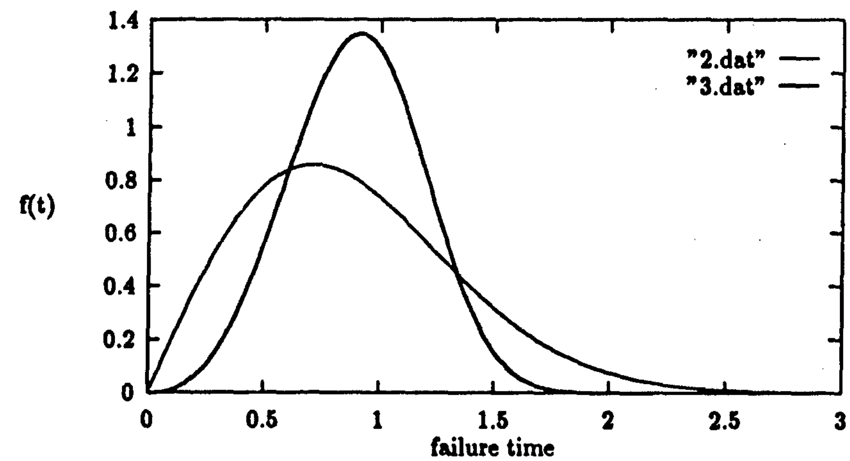

Figure 2. Graph of Weibull distributions: shape parameter k = 2.0 and 3.5

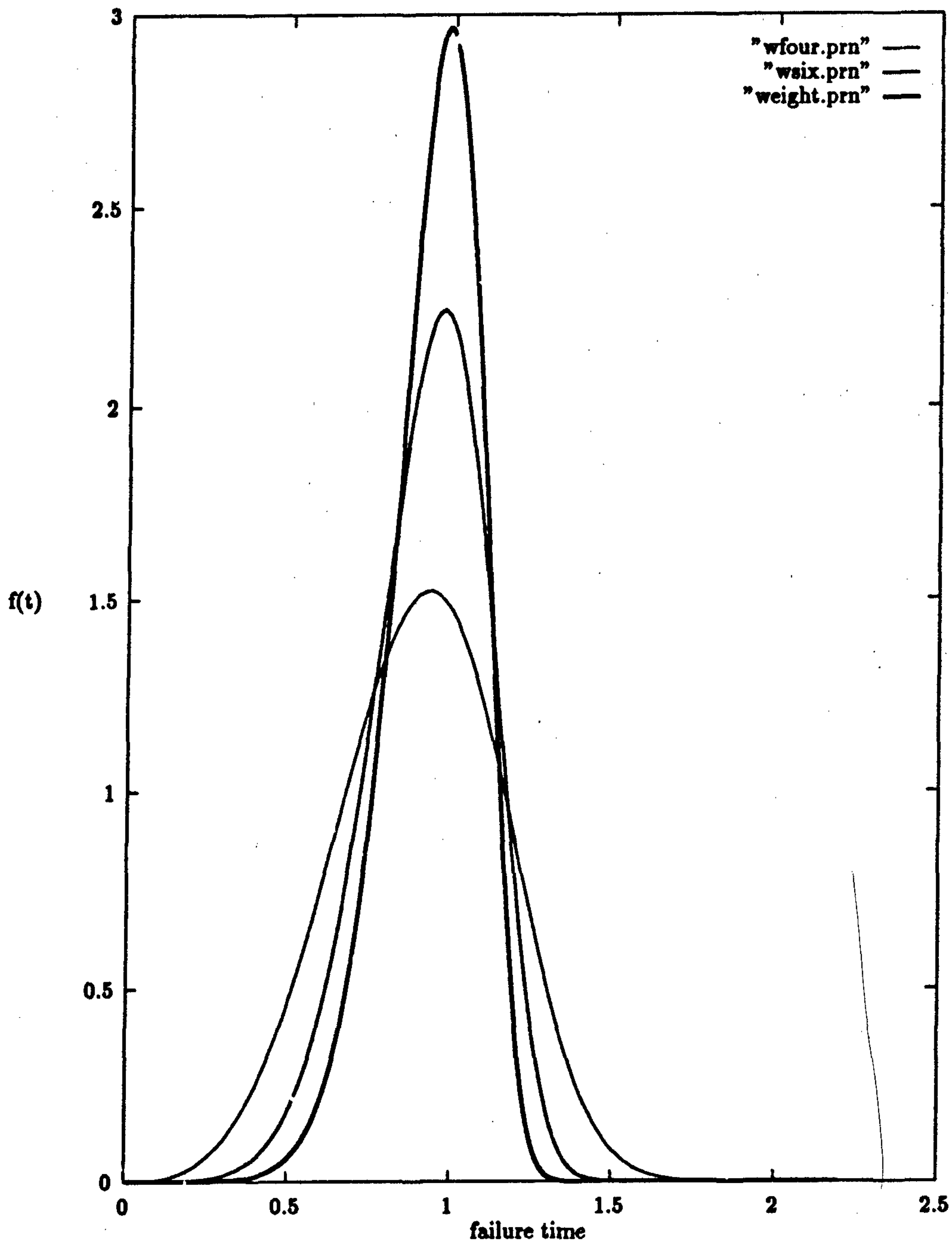

Figure 3. Graph of Weibull distributions: k = 4.0, 6.0, 8.0

2.3 Reliability

The reliability of a system is the probability that, when operating under stated environmental conditions, the system will perform its intended function adequately for a specified interval of time(7:1). Mathematically, the sum of the reliability function and the cumulative distribution function must always be one since they are complementary events. For the two parameter Weibull case, the reliability function is given by

$$R(x;\theta,\beta) = e^{-\left(\frac{x}{\theta}\right)^{\beta}} \tag{9}$$

which is easily seen to be $1 - F(x)$.

2.4 Classical Goodness-of-Fit Tests

In general, formal goodness-of-fit tests seek to determine whether a sample data set can be hypothesized to have come from a commonly known probability distribution. Since inferences about the general population are based on the smaller data set, it is important to test the fit of the data to the population.

2.4.1 Chi-squared Type Tests. In 1900, Karl Pearson abandoned the assumption that biological populations were normally distributed, introducing his system of distributions to provide other models. The need to test the fit of his proposed models arose from this, and today the Chi- squared test statistic is among the most used statistical procedures. Pearson's idea was to reduce the general problem of testing fit to a multinomial setting by basing a test on a comparison of observed cell counts versus their expected values under the hypothesis H_0 to be tested. This reduction of data to grouped sets discards some information, making tests of this type somewhat less powerful than other classes of tests of fit(8:431–432).

2.4.2 Empirical Distribution Function Type Tests. Goodness of fit tests based on the empirical distribution function(EDF) are discussed extensively by Stephens(19). A great number of statistics have been proposed for testing the null hypothesis H_0 based on the idea of measuring the distance between $F_0(x)$ and $F_n(x)$ where $F_n(x)$ is defined as the number of data points in a sample of size n that are less than x divided by the sample size n. Three widely accepted test statistics in this area are:

- Kolmogorov-Smirnov statistic (K-S)
- Cramer-von Mises statistic (W^2)
- Anderson-Darling statistic (A^2)

Tests based on these statistics are distribution free; they do not depend on the hypothesized distribution if all the parameters are completely specified(8:433–434).

2.5 Tests based on Order Statistics

The subject of order statistics deals with properties and applications of ordered random variables and of functions of these variables. When random variables $x_{(i)}$ are arranged in ascending order of magnitude, then $x_{(i;n)}$ is said to be the i^{th} order statistic in a sample of size n. To illustrate the application of order statistics to goodness of fit tests, consider a life test on a certain electrical component. A random selection of n specimens is made from the population of items available. This sample is placed *on test.* If the test is continued until all sample specimens have failed, the sample is said to be complete and it consists of the ordered observations of failure times: $x_{(1;n)}$ to $x_{(n;n)}$. In the common case when testing is terminated with survivors, the sample is said to be censored. Some frequently encountered functions of order statistics are:

- The extremes $x_{(1;n)}$ and $x_{(n;n)}$
- The range $W = x_{(n;n)} - x_{(1;n)}$
- The extreme deviate from the sample mean defined as $x_{(n;n)} - \mu$

These statistics play an important role in practical applications. The extremes arise in the study of floods and droughts, as well as in breaking strength and fatigue failure studies. The range is widely used by quality control practitioners to provide quick estimates of the standard deviation in a normal distribution(3:1–5).

2.6 Specific Tests for the Weibull Distribution

Mann, Scheuer and Fertig (1973) derived and published critical values for a test of fit of data to a two-parameter Weibull distribution with unknown parameters of scale and shape. Their test statistic S, is the ratio of a linear combination of the sum of order statistics divided by the expected values of a normalized sum of order statistics from a censored sample. It was shown to have higher power than the classical alternatives in many cases. The advantage of the S statistic is that it is relatively easy to calculate and the parameters of the Weibull distribution do not need to be known nor estimated(11).

Littell, McClave and Offen (1979) developed modified test statistics based on the Kolmogorov-Smirnov, Cramer von Mises and the Anderson-Darling statistics discussed earlier. They conducted an extensive power study of those three statistics and Mann's S statistic. Results were somewhat mixed with the the S statistic performing well against 3 of the 5 alternative distributions(9:257–266).

Tiku and Singh (1981) extended the pioneering work of Mann, Scheuer and Fertig in their development of a new test statistic closely related to the S statistic. Denoted by Z^*, this statistic is the ratio of sums of ordered observations divided

by sums of their expected values, with different limits on the summations when compared to S. Tiku and Singh incorporated the results of Mann's and Littell's power studies in their assessment of the Z^* test statistic. On the whole, they found their statistic more powerful than their prominent competitors. Due to the computational ease and the invariance in scale and location, it seems logical to use the Z^* test statistic over the modified EDF-based statistics developed by Littel and others(22:907–916).

III. METHODOLOGY

The goal of this thesis is to develop a new goodness-of-fit test for the Weibull distribution where the shape parameter β is known. The scale parameter θ and the location parameter δ do not need to be known nor estimated.

3.1 The Z^* Test Statistic

The test statistic used throughout this thesis will be a derivative of the one used by Tiku and Singh(22). It will be denoted Z^* and is given by

$$Z^* = \frac{2\sum_{i=1}^{n-2} (n-1-i)\, G_i}{(n-2)\sum_{i=1}^{n-1} G_i} \tag{10}$$

where G_i is the ratio defined by

$$G_i = \frac{X_{(i+1)} - X_i}{\mu_{(i+1:n)} - \mu_{(i:n)}} \tag{11}$$

The numerator of G_i is the difference between the i^{th} and the $(i+1)^{st}$ order statistic of the sample. This difference will be referred to as the i^{th} gap of the sample. There will be $n-1$ gaps in a sample of size n. The values of the order statistics, X, are the raw data values; not the log-transformed values.

In simplified form, the ratio G_i becomes

$$G_i = \frac{X_{(i+1)} - X_i}{Mudif_i} \tag{12}$$

The denominator of G_i is the expected value difference, denoted *Mudif*, between the i^{th} and the $(i+1)^{st}$ order statistic from the standard Weibull distribution with

scale parameter θ, shape parameter k and location parameter equal to zero. There will be $(n-1)$ *Mudif* values in a sample of size n. The μ_i are obtained from the expected value formula given by

$$E(x_{m,n};\theta,k) = \theta n \binom{n-1}{m-1} \Gamma\left(1+\frac{1}{k}\right) \sum_{j=0}^{m-1} \binom{m-1}{j} \frac{-1^{(m+j-1)}}{(n-j)^{(1+\frac{1}{k})}} \tag{13}$$

which was obtained from H. Leon Harter's referenced text(5). Tabled values of $Mudif_i$ for sample sizes 5 to 35 and shape parameters 0.5, 1.0 and 1.5 are found in Appendix D.

3.2 Computation of Critical Values for Z^*

This thesis will utilize the Monte Carlo method for obtaining the critical values of the Z^* test statistic. The procedures used are detailed in the following steps:

1. For a fixed sample size n and shape parameter k, random deviates from the Weibull distribution are generated using the IMSL subroutine **RNWIB**. All Weibull deviates are generated with scale parameter equal to one and location parameter equal to zero.

2. The n random Weibull deviates were additively scaled by ten.

3. The n scaled Weibull deviates were sorted in ascending order using IMSL subroutine **SVRGN**. The values obtained after the sort operation are the X_i values used in the numerator of G_i.

4. The expected value differences μ_i were obtained separately and read into the FORTRAN program for use in the denominator of G_i.

5. The G_i values were input into the summation formula to compute the Z^* test statistic value for this one sample of size n.

6. Steps 1 to 5 were repeated 10,000 times, thus generating 10,000 independent Z^* statistics.

7. The 10,000 Z^* statistics are sorted in ascending order using the IMSL subroutine **SVRGN**.

8. The 80th, 85th, 90th, 95th and the 99th percentile were found by linear interpolation. These percentiles comprise the critical values for the test statistic.

The process was repeated for different sample sizes and shape parameters. For shape parameters $k = 0.5$, 1.0, 1.5, sample size ranged from 5 to 35 in multiples of 5. For shape parameters $k = 2.0$, 3.0, 4.0, sample size was limited to 5, 10, 15 and 20. In addition, two runs were made at a sample size of 20 for shape parameters $k = 6.0$ and $k = 8.0$. The output of the analysis was arranged in tabular form and can be found in Appendix A.

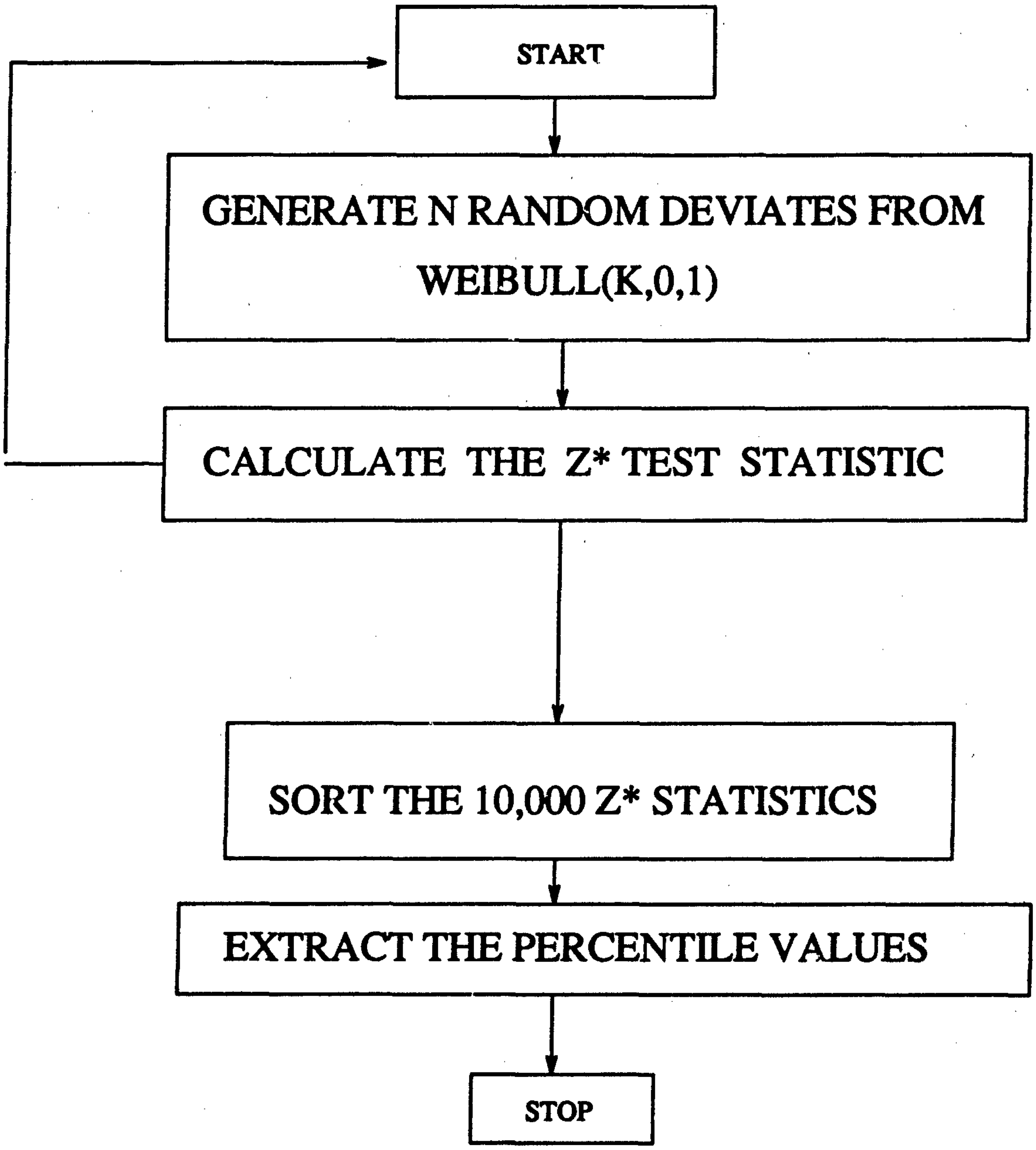

Figure 4. Generation of Critical Values for Z^*

3.3 Power Study of the Z^* Statistic

The benchmark for any goodness-of-fit test statistic is the *Power of the Test.* It is defined as the probability of rejecting the null hypothesis, H_0, when an alternate hypothesis H_a is true. In this thesis, nine alternate distributions are utilized in the power study for the Z^* test statistic. A direct comparison with the prominent competitors was made for shape = 1.0 and sample size 5, 15 and 25. Power figures for this case were obtained from prior AFIT theses by Cortes(4) and Bush(2). Power data for all other combinations of shape parameter k and sample size n were accomplished independently via the Monte Carlo method.

3.3.1 The Distributions H_0 and H_a. The null hypothesis for the power study was that sample deviates follow a Weibull distribution with shape parameter k. The values of k used were k = 0.5, 1.0, 1.5, 2.0, 3.0, 4.0, 6.0 and 8.0. The 9 alternative hypotheses, H_a, are summarized in the accompanying table. They are identified by Distribution numbers 2 thru 10. Graphical representations of H_0 versus H_a can be found in Appendix C.

Table 1. Parametric Distribution Functions Utilized throughout this Thesis

Distribution	Identification:Parameters
1a	Weibull with shape parameter = 0.5
1b	Weibull with shape parameter = 1.0
1c	Weibull with shape parameter = 1.5
2	Weibull with shape parameter = 2.0
3	Weibull with shape parameter = 3.5
4	Gamma with shape parameter = 1.0
5	Gamma with shape parameter = 2.0
6	Uniform with a= 0.0 and b= 1.0
7	Normal with mean= 0.0 and stdev= 1.0
8	Beta with P=2.0 and Q=3.0
9	Beta with P=2.0 and Q=2.0
10	Beta with P=1.0 and Q=1.0

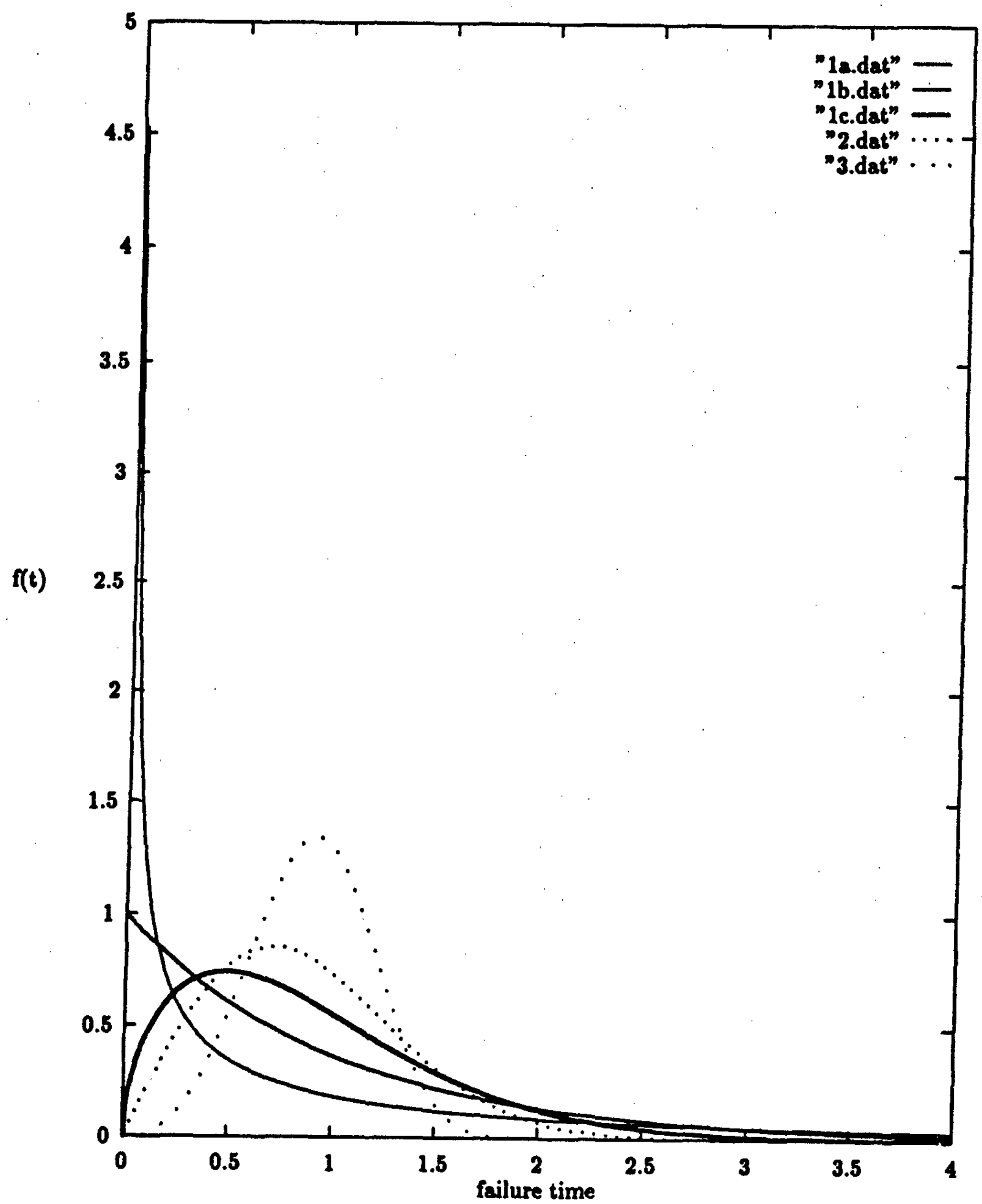

Figure 5. Graph of Weibull distributions: k = 0.5, 1.0, 1.5, 2.0 and 3.5

3.3.2 Power Study Process. The power study process was very similar to the critical value determination procedure. A Monte Carlo simulation was accomplished with 5000 iterations and a different random number seed than what was used in the critical value program. The steps in the process were:

1. Random deviates from H_a for a fixed sample size n are generated using the appropriate IMSL subroutine.

2. Deviates were additively scaled by 10.0 and sorted in ascending order. The values obtained at this point are the X_i values used in the numerator of G_i.

3. The expected value differences for a sample of size n, shape k and scale parameter equal to one are read into the program to comprise the denominator of G_i.

4. The Z^* test statistic is calculated for each iteration.

5. The Z^* value attained is compared to the critical value at each of five α levels.

6. The number of times the Z^* statistic exceeds the critical value is counted for each of the five α levels. Exceeding the critical value is equivalent to rejecting H_0 at that significance level.

The process was repeated for the different alternate distributions, H_a, at various sample sizes. The output of this portion of the analysis can be found in Appendix B. The direct comparison with other test statistics will be discussed in Chapter 4.

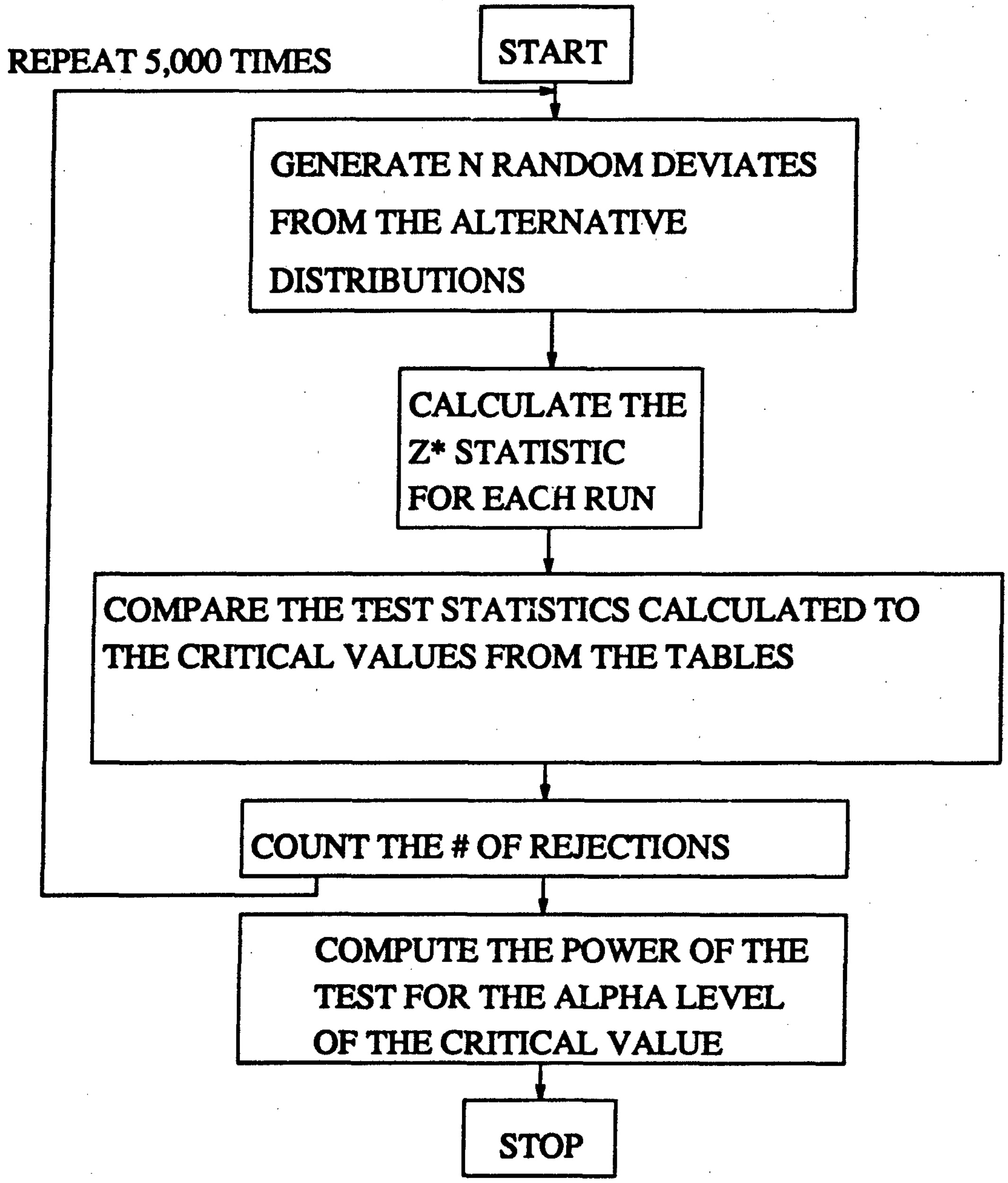

Figure 6. Generation of Power tables for the Z^* test statistic

3.4 *Analysis of the Relationship between Critical Values and Sample Size*

The critical values for the Z^* test statistic were computed at sample sizes in multiples of five. For certain shape parameters, a relationship was postulated between critical values and sample size. This relationship would enable critical values to be computed for any sample size *within* the range 5 to 35 with reasonable accuracy. A linear regression analysis was accomplished using a relationship between critical value, cv and sample size n:

$$cv = a_0 + a_1 n + a_2 log(n) \tag{14}$$

Results of this regression analysis are also presented in Chapter 4.

IV. FINDINGS

This chapter discusses the findings of this research effort. Results applicable to each of the objectives set forth in Chapter 1 are presented in sequence.

4.1 The Z^* Test Statistic

The Z^* statistic was utilized during all phases of this thesis effort. On the whole, it was relatively simple to compute once the necessary elements of its composition were available. The only problem I encountered was the inclusion of the expected value differences in the denominator of G_i. The formula obtained from H. Leon Harter's reference tables(5:14) was input into a MATHCAD program for calculation of the expected values. The program worked well from samples size 5 thru 35 in jumps of five. However, when $n = 40$ was tried the equation exceeded the limits of the program. Values obtained prior to that point were crosschecked against the tabled values found in Harter's book. In all cases, there was total agreement. For this reason, the sample sizes of this thesis effort were capped at $n = 35$.

4.2 Critical Values for the Z^* Test Statistic

Critical values for Z^* were obtained via Monte Carlo method with 10,000 iterations. Linear interpolation at the appropriate levels of significance yielded the desired numerical values of Z^*. For the sake of discussion, consider the critical values obtained for shape parameter k = 1.0: Obviously, as α goes from 0.20 to 0.01 the critical value increases as it must.

Table 2. Critical Values for Z* test statistic: Samplesize N, shape parameter 1.0, α levels are 0.20 thru 0.01

N	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	1.0	1.290	1.362	1.443	1.558	1.740
10	1.0	1.170	1.210	1.262	1.336	1.466
15	1.0	1.135	1.169	1.207	1.264	1.369
20	1.0	1.115	1.140	1.175	1.223	1.311
25	1.0	1.100	1.123	1.151	1.196	1.277
30	1.0	1.091	1.112	1.137	1.176	1.255
35	1.0	1.087	1.106	1.131	1.167	1.233

Within each α level, the critical values decrease monotonically as the sample size is increased from 5 to 35. However, note that the rate of decrease within each α level is not linear. This trend was noted for all shape parameters considered in this thesis. Tabled values for all shapes considered are found in Appendix A.

4.3 Power Study of Z^* Test Statistic

The power of a goodness-of-fit test is the probability of rejecting the null hypothesis, H_0, when it is false. Therefore, we seek power values as close to 1.0 as possible for any distribution that is not the null distribution. Nine alternate distributions were utilized for the power study of Z^* via Monte Carlo method and 5000 iterations.

4.3.1 Cross-checks. I would like to point out three crosschecks I used throughout the computation phases of this effort:

- When the null hypothesis is input into the power study computer program, the attained power should be the claimed level of significance, α. In all cases, power attained when H_0 was true was within 5 percent of the α level.

- Alternate distribution number 4 was the Gamma distribution with shape parameter equal to one. This distribution is identical to the Weibull distribution with shape parameter equal to one provided the scale and location parameters are also the same. Thus, power values for Distribution 4 when the null hypothesis is Weibull with shape 1.0 should approach the claimed level of significance.
- Alternate distribution number 6 is equivalent to alternate distribution number 10. The beta distribution with parameters (1,1) is the same as the standard uniform distribution. Therefore, power values for any null hypothesis H_0 should be essentially the same for these two equivalent alternatives.

Power study results for the Z^* test statistic were mixed. For shape parameters less than two, the test yielded good to excellent results. For shape parameter two and higher, the test results were poor. These results were not totally unexpected.

4.3.2 Analysis of Skewness. In 1980, Tiku concluded that the Z^* test statistic had good power properties for:

- Testing the exponential and gamma distributions against skew and symmetric alternatives
- Testing the uniform, normal and logistic distributio. against alternative distributions that were skewed(20:271).

Table 3. Skewness of the Weibull Distribution as a function of the Shape Parameter

Shape Parameter	Skewness
0.5	6.619
1.0	2.000
1.5	1.072
2.0	0.631
3.0	0.168
3.5	0.025
4.0	-0.087
5.0	-0.254
6.0	-0.373
10.0	-0.637
20.0	-0.860

Based on my power study results and Tiku's conclusions, it seemed logical to look at the skewness values for the null hypotheses. Since the Weibull distribution can be both symmetric and skew depending on the value of the shape parameter, the two situations described by Tiku do occur. Skewness values for the Weibull distribution were obtained from the text by Kapur and Lamberson(7). A symmetric distribution will have a skewness value of zero. However, departures from symmetry are slight for skewness values between -1 and +1. Thus, when the shape parameter of the Weibull is two or greater the skewness is in the range between -1 and +1. These facts are borne out by the graphical depictions of the Weibull for both the symmetric and skew cases.

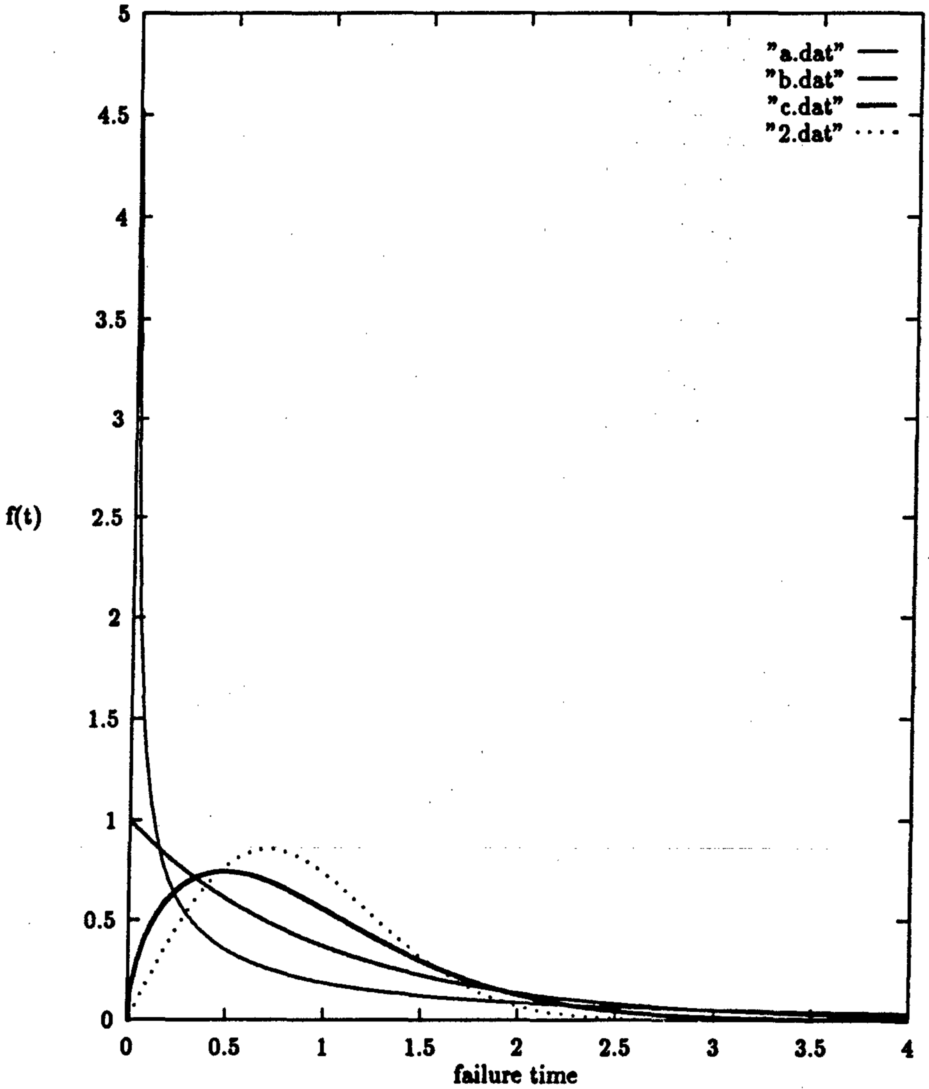

Figure 7. Graph of Weibull distributions: k = 0.5, 1.0, 1.5, 2.0

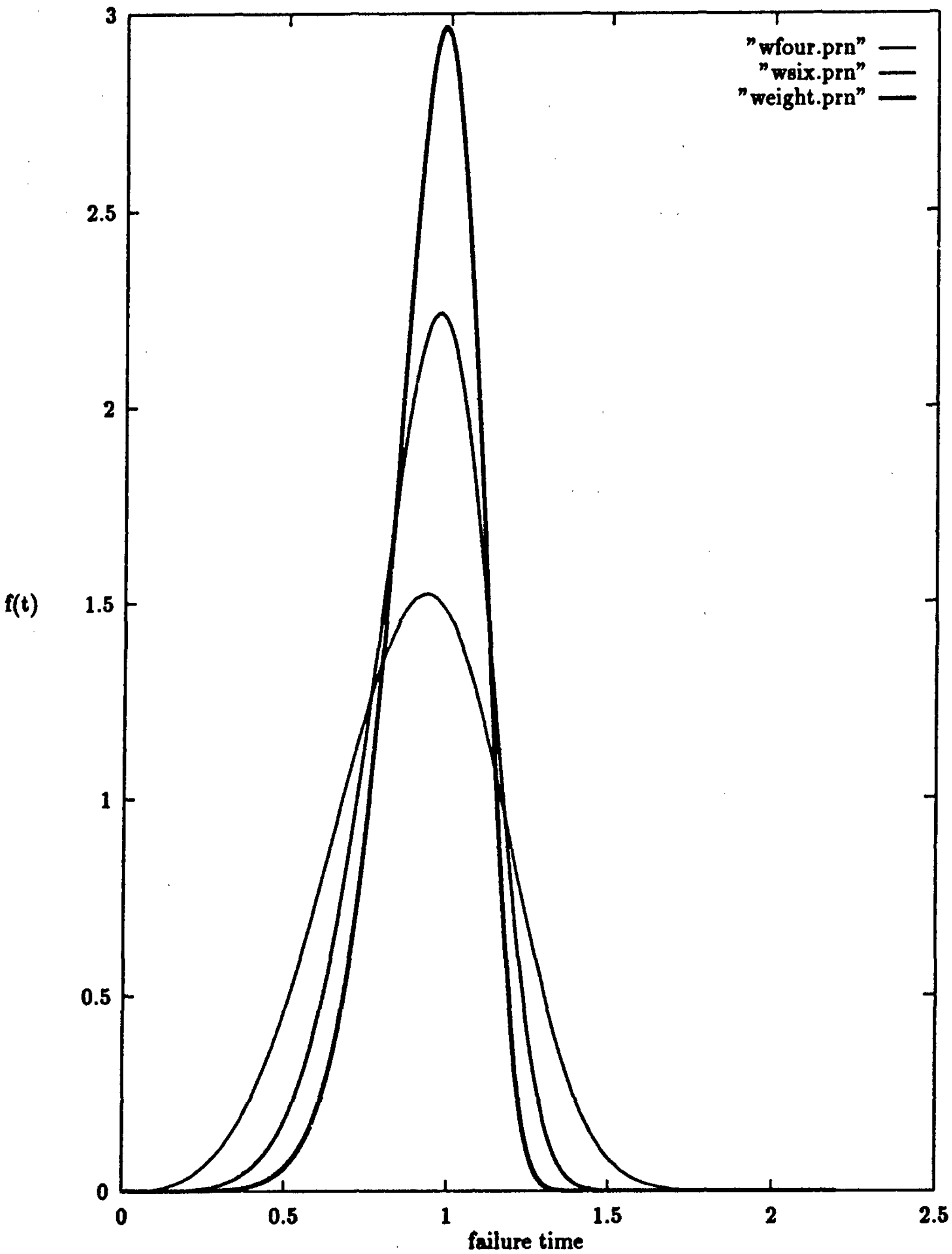

Figure 8. Graph of Weibull distributions: k = 4.0, 6.0, 8.0

4.4 *Power Results for the Z^* Test Statistic*

In the following sections, the power results are discussed in four phases due to the differing levels of success attained by this analysis. As the null hypothesis H_0 changes from being highly skewed to symmetric, the power results change from excellent to poor. Of course, larger sample sizes produce better power at all values of the null hypothesis. The four groupings for discussion are:

- Weibull shape parameter 0.5
- Weibull shape parameter 1.0
- Weibull shape parameter 1.5
- Weibull shape parameters 2.0, 3.0, 4.0, 6.0 and 8.0

For the purposes of subsequent discussion, I subjectively chose a scale of power values as defined here:

- Power of the test values of 0.90 or greater are excellent
- Power of the test values of 0.70 to 0.89 are good
- Power of the test values of 0.50 to 0.69 are fair
- Power of the test values below 0.50 are poor

Although this scale is subjective, the broad scope of the data I accumulated demanded some aggregation for the sake of clarity.

4.4.1 Power Results for shape parameter k = 0.5. The Weibull distribution with shape parameter 0.5 is the most highly skewed distribution considered in this analysis. As indicated on the accompanying plots, it demonstrates the lowest level of correlation with the other distributions. As such, the goodness-of-fit test at this shape parameter should produce the best results. This assertion is quite valid as demonstrated by the power values attained via the Monte Carlo method. At sample sizes of 10 or greater, the power of the test is good to excellent. For the sake of illustration, the power study data is presented for sample size n = 10. All power tables from sample size 5 through 35 are given in Appendix B. At a significance level

Table 4. Power of the Test: Samplesize N=10, shape parameter K=0.5, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.199	0.152	0.105	0.053	0.011
2	0.986	0.977	0.951	0.884	0.618
3	0.997	0.992	0.984	0.948	0.771
4	0.876	0.822	0.724	0.549	0.218
5	0.968	0.943	0.897	0.787	0.449
6	0.989	0.978	0.952	0.874	0.569
7	0.997	0.993	0.981	0.950	0.782
8	0.992	0.985	0.965	0.903	0.641
9	0.996	0.991	0.977	0.929	0.712
10	0.986	0.970	0.944	0.865	0.576

of 0.05, the power of the test is at least good for 8 of the 9 alternate hypotheses H_a. For α levels of 0.10 and higher, the power is good to excellent for all 9 of the H_a. These were the highest power values attained for the Z^* test statistic.

4.4.2 Power Results for shape parameter k = 1.0. This section will contain the most comprehensive analysis since the discussion will focus on both the Monte Carlo results and a comparison with competitive goodness-of-fit test statistics.

4.4.2.1 Monte Carlo Results. The Weibull distribution with shape parameter k = 1.0 is a non-symmetric distribution with skewness value of two. It is equivalent to the negative exponential distribution as well as the Gamma distribution with shape parameter equal to one. It is much wider in appearance than the highly skewed Weibull distribution with shape parameter k = 0.5 as indicated on the plot.

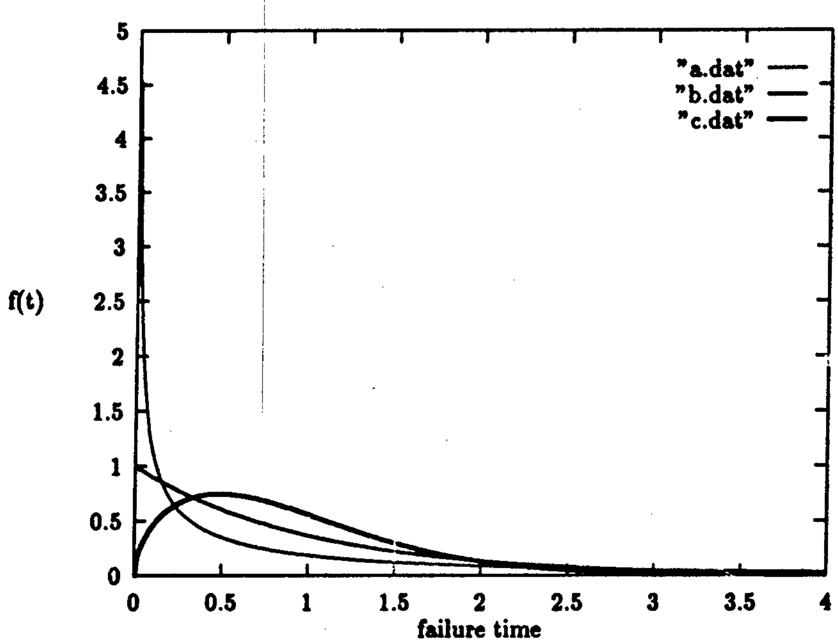

Figure 9. Graph of Weibull distributions: shape parameter k = 0.5, 1.0, 1.5

Power results vary depending on the significance level chosen. For α levels of 0.10 or higher, good to excellent power is attained at sample sizes of 15 or greater. At the typical significance level of 0.05, sample sizes of 20 or higher are required to achieve good to excellent power values. The table for sample size 20 is presented here; all other power tables for this shape parameter are found in Appendix B.

Table 5. Power of the Test: Samplesize N=20, shape parameter K=1.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.198	0.150	0.096	0.051	0.011
2	0.940	0.912	0.865	0.763	0.502
3	0.993	0.987	0.976	0.946	0.834
4	0.196	0.154	0.103	0.055	0.012
5	0.661	0.593	0.485	0.336	0.129
6	0.958	0.934	0.889	0.794	0.535
7	0.991	0.986	0.978	0.951	0.861
8	0.969	0.955	0.914	0.834	0.588
9	0.990	0.982	0.964	0.922	0.759
10	0.958	0.937	0.893	0.806	0.545

As previously stated, good to excellent power values are achieved at significance levels of 0.05 or greater. Note that distribution number 4 is the same as the null hypothesis(distribution 1) and its attained power is approximately the significance level α as expected.

4.4.2.2 Comparison of Z^* and its Competition. Power data for the prominent competition in a goodness-of-fit test for the Weibull(shape = 1.0) was available in a prior AFIT thesis done by Bush(2). Values for the power are limited to sample sizes of 5, 10 and 15 at alpha levels of 0.05 and 0.01. The nine alternate hypotheses used in my thesis were chosen to coincide with the prior results making direct comparison possible. Since distribution 4 is essentially equivalent to the null hypothesis, the power of the test for distribution 4 should be the α value. This occurs in all cases with minor variability due to the random nature of the Monte Carlo method. Therefore, subsequent analysis will involve only 8 alternate hypotheses H_a. The three competing test statistics are the:

- Kolmogorov-Smirnov (K-S)
- Cramer von Mises (W^2)
- Anderson-Darling (A^2)

For sample size 5 and alpha levels of 0.05 and 0.01, the comparison is summarized in the accompanying tables. Although I do not consider this to be a satisfactory value of the power, the power values of Z^* are higher than its competitors in 47 out of 48 direct comparisons. The value of 48 comes from 8 alternate hypotheses, 3 competitive test statistics and two alpha levels.

Table 6. Comparative Power Study of Z* and Competing Alternative Test Statistics: Sample size 5, shape parameter 1.0, alpha level 0.05

Distribution	Z^*	K-S	W^2	A^2
H_0True	.047	.045	.045	.049
Weibull(k=2)	.137	.051	.055	.049
Weibull(k=3.5)	.226	.079	.097	.052
Gamma(shape=1.0)	.049	.047	.047	.045
Gamma(shape=2.0)	.088	.040	.043	.114
Uniform(0,1)	.188	.072	.084	.050
Normal(0,1)	.218	.079	.099	.202
Beta(p=2,q=3)	.161	.057	.066	.014
Beta(p=2,q=2)	.200	.069	.082	.031
Beta(p=1,q=1)	.186	.069	.085	.048

Table 7. Comparative Power Study of Z* and Competing Alternative Test Statistics: Sample size 5, shape parameter 1.0, alpha level 0.01

Distribution	Z^*	K-S	W^2	A^2
H_0True	.011	.009	.008	.007
Weibull(k=2)	.033	.003	.002	.004
Weibull(k=3.5)	.056	.007	.006	.007
Gamma(shape=1.0)	.010	.011	.011	.007
Gamma(shape=2.0)	.021	.004	.003	.017
Uniform(0,1)	.054	.007	.005	.006
Normal(0,1)	.062	.003	.004	.061
Beta(p=2,q=3)	.038	.003	.002	.001
Beta(p=2,q=2)	.051	.005	.005	.004
Beta(p=1,q=1)	.055	.005	.005	.006

For sample size 15 and alpha levels of 0.05 and 0.01, the comparison becomes more favorable for Z^*. It exceeds the power of its competition in 48 of 48 direct comparisons. This higher sample size of 15 still does not attain acceptable power values according to the scale of *good to excellent* discussed previously at the beginning of this section.

Table 8. Comparative Power Study of Z* and Competing Alternative Test Statistics: Sample size 15, shape parameter 1.0, alpha level 0.05

Distribution	Z^*	K-S	W^2	A^2
H_0True	.048	.049	.053	.043
Weibull(k=2)	.581	.277	.346	.321
Weibull(k=3.5)	.852	.568	.667	.624
Gamma(shape=1.0)	.051	.050	.053	.050
Gamma(shape=2.0)	.240	.099	.116	.122
Uniform(0,1)	.650	.328	.446	.384
Normal(0,1)	.860	.606	.699	.685
Beta(p=2,q=3)	.672	.340	.428	.359
Beta(p=2,q=2)	.791	.454	.578	.508
Beta(p=1,q=1)	.645	.323	.434	.363

Table 9. Comparative Power Study of Z* and Competing Alternative Test Statistics: Sample size 15, shape parameter 1.0, alpha level 0.01

Distribution	Z^*	K-S	W^2	A^2
H_0True	.009	.012	.012	.009
Weibull(k=2)	.307	.105	.139	.107
Weibull(k=3.5)	.640	.315	.422	.347
Gamma(shape=1.0)	.007	.012	.012	.009
Gamma(shape=2.0)	.066	.029	.027	.021
Uniform(0,1)	.359	.125	.191	.138
Normal(0,1)	.658	.362	.465	.434
Beta(p=2,q=3)	.375	.134	.177	.122
Beta(p=2,q=2)	.528	.199	.291	.216
Beta(p=1,q=1)	.352	.117	.178	.122

For sample size 25 and alpha levels of 0.05 and 0.01, the comparison remains in favor of Z^*. The power attained by the Z^* test statistic exceeds that of the competition in 48 of 48 direct comparisons. At this sample size of 25, the power values achieved fall into the good to excellent range.

Table 10. Comparative Power Study of Z* and Competing Alternative Test Statistics: Sample size 25, shape parameter 1.0, alpha level 0.05

Distribution	Z^*	K-S	W^2	A^2
H_0True	.052	.057	.056	.047
Weibull(k=2)	.877	.575	.730	.655
Weibull(k=3.5)	.987	.885	.947	.930
Gamma(shape=1.0)	.052	.048	.055	.044
Gamma(shape=2.0)	.435	.196	.245	.231
Uniform(0,1)	.892	.575	.746	.703
Normal(0,1)	.984	.904	.950	.938
Beta(p=2,q=3)	.926	.628	.773	.715
Beta(p=2,q=2)	.974	.771	.899	.867
Beta(p=1,q=1)	.894	.563	.737	.694

Table 11. Comparative Power Study of Z* and Competing Alternative Test Statistics: Sample size 25, shape parameter 1.0, alpha level 0.01

Distribution	Z^*	K-S	W^2	A^2
H_0True	.010	.012	.011	.009
Weibull(k=2)	.659	.324	.424	.351
Weibull(k=3.5)	.933	.717	.828	.781
Gamma(shape=1.0)	.010	.010	.011	.009
Gamma(shape=2.0)	.190	.069	.083	.062
Uniform(0,1)	.684	.303	.449	.361
Normal(0,1)	.941	.756	.857	.819
Beta(p=2,q=3)	.748	.351	.506	.406
Beta(p=2,q=2)	.889	.524	.695	.618
Beta(p=1,q=1)	.677	.292	.442	.151

4.4.3 Power Results for shape parameter $k = 1.5$. The Weibull distribution with shape parameter k = 1.5 is not a highly skewed distribution. It has a skewness value of 1.072 putting it in the transition zone from skewness to symmetry. Satisfactory values of power are attained at high sample sizes of 30 and 35 for significance levels greater than or equal to 0.10. At α levels of 0.05 and 0.01, the test attains good results in only 3 of 9 alternate hypotheses. However, the sample sizes necessary for a satisfactory goodness-of-fit test are not prohibitively large. The table for sample size 30 is presented below with all tables appearing in Appendix B.

Table 12. Power of the Test: Samplesize N=30, shape parameter K=1.5, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.198	0.147	0.099	0.046	0.008
2	0.629	0.548	0.446	0.317	0.116
3	0.958	0.939	0.909	0.854	0.667
4	0.005	0.003	0.002	0.001	0.000
5	0.112	0.081	0.050	0.025	0.005
6	0.851	0.801	0.716	0.586	0.304
7	0.959	0.941	0.911	0.855	0.677
8	0.800	0.725	0.637	0.481	0.213
9	0.947	0.915	0.866	0.769	0.507
10	0.851	0.787	0.700	0.572	0.305

4.4.4 Power Results for shape parameter k = 2, 3, 4, 6 and 8. The Weibull distribution with shape parameters in the range from 2 through 8 have skewness values in the interval from 0.63 to -0.53. At a value of approximately 3.7, the skewness is zero signifying total symmetry. The Z^* test statistic did a poor job of discriminating between the null hypothesis H_0 and the alternate hypothesis H_a in almost all cases considered. The Weibull with shape two has the highest skewness value of this group and the power table for sample size 20 is presented. Note that even when a significance level of 0.20 is considered, the power does not ever exceed 0.70 (our subjective benchmark).

Table 13. Power of the Test: Samplesize N=20, shape parameter K=2.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.199	0.144	0.094	0.050	0.009
2	0.197	0.147	0.096	0.051	0.011
3	0.630	0.558	0.474	0.338	0.140
4	0.003	0.002	0.000	0.000	0.000
5	0.026	0.019	0.011	0.005	0.000
6	0.491	0.407	0.313	0.197	0.064
7	0.650	0.590	0.512	0.383	0.178
8	0.320	0.249	0.174	0.092	0.022
9	0.578	0.492	0.378	0.249	0.078
10	0.494	0.411	0.315	0.193	0.061

The power table for shape parameter 8 enforces my assertion of unsatisfactory performance at any α level.

Table 14. Power of the Test: Samplesize N=20, shape parameter K=8.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.196	0.141	0.092	0.049	0.010
2	0.004	0.003	0.001	0.000	0.000
3	0.039	0.024	0.012	0.004	0.000
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.057	0.037	0.019	0.005	0.000
7	0.047	0.032	0.018	0.009	0.001
8	0.008	0.003	0.001	0.000	0.000
9	0.031	0.016	0.009	0.002	0.000
10	0.055	0.037	0.020	0.008	0.001

4.5 *Relationship between Critical Values and Sample Size*

The analyses conducted throughout this research effort were done at sample sizes in multiples of five. Thus, critical values are only tabulated at these multiples. In order to extend the applicability of this goodness-of-fit test, a relationship between critical value and sample size was *fit* to allow usage between 5 and 35 at any value of sample size n. A regression analysis was conducted using the **SX** statistical package with very satisfactory results:

Table 15. Critical Values expressed as an approximate function of sample size n: $cv = a_0 + a_1 n + a_2 log_{10}\, n$ (for critical values at $\alpha = 0.05$)

shape	a_0	a_1	a_2	R^2
0.5	2.11927	0.00951	-0.77293	0.9884
1.0	2.07592	0.01083	-0.82935	0.9917
1.5	2.07045	0.01152	-0.85551	0.9887

This regression analysis was limited to the shape parameters of 0.5, 1.0 and 1.5 since the test statistic performed satisfactorily in that range only.

4.6 A Sample Problem Using the Z^* Test Statistic

The following sample problem will demonstrate the simplicity of using the Z^* test statistic. Twenty data points are generated from the negative exponential distribution. They are contained in the file yvals.dat. The expected value differences, $(Mudif)_i$, are read from the file d_n20_k1.prn for use in the denominator of G_i.

$$mudif := READPRN\left[d\_n20\_k1_{prn}\right] \qquad x := READPRN\left[yvals_{dat}\right]$$

$$n := 20 \qquad j := 1 \,..n - 2$$

$$i := 1 \,..19 \qquad k := 1 \,..n - 1$$

$$xx := sort(x) \qquad gap_i := xx_{i+1} - xx_i$$

$$g_i := \frac{gap_i}{mudif_i} \qquad zstar := \frac{2 \cdot \sum_j (n - 1 - j) \cdot g_j}{(n - 2) \cdot \sum_k g_k}$$

$$zstar = 1.080493$$

$$x = \begin{bmatrix} 0.68901 \\ 1.078596 \\ 0.505093 \\ 1.738269 \\ 2.507165 \\ 0.284493 \\ 0.661113 \\ 2.809705 \\ 1.810783 \\ 0.963443 \\ 1.802222 \\ 4.513602 \\ 1.679691 \\ 1.034693 \\ 2.092446 \\ 0.312935 \\ 0.085878 \\ 0.123732 \\ 0.051568 \\ 1.304105 \end{bmatrix} \qquad xx = \begin{bmatrix} 0.051568 \\ 0.085878 \\ 0.123732 \\ 0.284493 \\ 0.312935 \\ 0.505093 \\ 0.661113 \\ 0.68901 \\ 0.963443 \\ 1.034693 \\ 1.078596 \\ 1.304105 \\ 1.679691 \\ 1.738269 \\ 1.802222 \\ 1.810783 \\ 2.092446 \\ 2.507165 \\ 2.809705 \\ 4.513602 \end{bmatrix} \qquad mudif = \begin{bmatrix} 0.052632 \\ 0.055556 \\ 0.058824 \\ 0.0625 \\ 0.066667 \\ 0.071429 \\ 0.076923 \\ 0.083333 \\ 0.090909 \\ 0.1 \\ 0.111111 \\ 0.125 \\ 0.142857 \\ 0.166667 \\ 0.2 \\ 0.25 \\ 0.333333 \\ 0.5 \\ 1 \end{bmatrix}$$

The Z^* test statistic is calculated from the two input files and compared to the critical value for determination of whether to accept or reject H_0.

The appropri: te critical values for sample size 20 and shape parameter 1.0 are given by

Table 16. Critical Values for Z* test statistic: Sample size 20, shape parameter 1.0, alpha levels are 0.20 thru 0.01

N	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
20	1.0	1.115	1.140	1.175	1.223	1.311

Since the value of Z^* we attained, 1.080493, was less than the critical value at any of the five α levels considered, we do not reject the null hypothesis that the sample came from the negative exponential distribution.

V. CONCLUSIONS AND RECOMMENDATIONS

The following conclusions and recommendations are based on the findings obtained during this research effort.

5.1 Conclusions

The following conclusions are made based on the critical value and power study programs accomplished during this thesis:

1. The tabled critical values of Z^* for the Weibull distribution with shape parameter known are valid. In the Monte Carlo simulations, the goodness-of-fit test achieved the claimed level of significance when the null hypothesis is true.
2. The power of the Z^* test statistic is poor when the null hypothesis H_0 is from a symmetric distribution. Specifically, when the shape parameter is 2.0 or more, the power of the test is unsatisfactory at sample sizes: 5, 10, 15 and 20.
3. The power of the Z^* test statistic is good to excellent in the following situations where H_0 is from a skewed distribution:
   - For shape parameter k = 0.5, the goodness-of-fit test has good to excellent power at sample sizes of ten or more.
   - For shape parameter k = 1.0, the goodness-of-fit test has good to excellent power at sample sizes of twenty or more.

- For shape parameter k = 1.5, the goodness-of-fit test has good to excellent power at sample sizes of thirty or more.

4. For shape parameter k = 1.0, the negative exponential distribution, this new test statistic, Z^*, achieves more power than any other current test statistic.
5. Critical values for Z^* at sample sizes not explicitly computed in this thesis can be approximated with the linear relationship postulated. High values of R^2 were attained with a relatively simple fit equation.

5.2 Recommendations

The following recommendations are proposed for further study:

1. The alternate distributions H_a considered in this thesis were from symmetric distributions in 8 of 9 cases. Suggest an increase in the number of skew distributions utilized in the power study of the Z^* test statistic.
2. The sample sizes considered in this thesis ranged from 5 to 35 in multiples of five. Suggest a more complete analysis be accomplished since the FORTRAN programs I wrote can be used again with minor revisions.
3. The data used throughout this thesis was assumed to come from complete samples. The luxury of a complete sample is not always available nor fiscally possible suggesting an analysis based on censored samples be undertaken.

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Table 17. Critical Values for Z* test statistic: Samplesize N, shape parameter 0.5, α levels are 0.20 thru 0.01

N	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	0.5	1.337	1.415	1.516	1.635	1.815
10	0.5	1.204	1.258	1.324	1.418	1.583
15	0.5	1.177	1.220	1.276	1.360	1.504
20	0.5	1.151	1.192	1.242	1.315	1.446
25	0.5	1.138	1.178	1.217	1.280	1.414
30	0.5	1.126	1.160	1.202	1.263	1.371
35	0.5	1.123	1.155	1.194	1.253	1.358

Table 18. Critical Values for Z* test statistic: Samplesize N, shape parameter 1.0, α levels are 0.20 thru 0.01

N	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	1.0	1.290	1.362	1.443	1.558	1.740
10	1.0	1.170	1.210	1.262	1.336	1.466
15	1.0	1.135	1.169	1.207	1.264	1.369
20	1.0	1.115	1.140	1.175	1.223	1.311
25	1.0	1.100	1.123	1.151	1.196	1.277
30	1.0	1.091	1.112	1.137	1.176	1.255
35	1.0	1.087	1.106	1.131	1.167	1.233

Table 19. Critical Values for Z* test statistic: Samplesize N, shape parameter 1.5, α levels are 0.20 thru 0.01

N	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	1.5	1.292	1.353	1.434	1.539	1.721
10	1.5	1.160	1.201	1.244	1.308	1.435
15	1.5	1.125	1.153	1.188	1.239	1.334
20	1.5	1.104	1.127	1.155	1.197	1.276
25	1.5	1.087	1.107	1.132	1.172	1.241
30	1.5	1.081	1.101	1.124	1.156	1.219
35	1.5	1.073	1.091	1.112	1.142	1.201

Table 20. Critical Values for Z* test statistic: Samplesize N, shape parameter 2.0, α levels are 0.20 thru 0.01

N	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	2.0	1.287	1.345	1.426	1.533	1.711
10	2.0	1.156	1.194	1.237	1.298	1.418
15	2.0	1.119	1.147	1.182	1.229	1.321
20	2.0	1.100	1.122	1.150	1.189	1.265

Table 21. Critical Values for Z* test statistic: Samplesize N, shape parameter 3.0, α levels are 0.20 thru 0.01

N	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	3.0	1.278	1.341	1.419	1.531	1.698
10	3.0	1.155	1.190	1.230	1.289	1.409
15	3.0	1.117	1.144	1.178	1.226	1.314
20	3.0	1.098	1.121	1.148	1.186	1.260

Table 22. Critical Values for Z* test statistic: Samplesize N, shape parameter 4.0, α levels are 0.20 thru 0.01

N	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
5	4.0	1.275	1.339	1.417	1.529	1.698
10	4.0	1.154	1.188	1.231	1.289	1.407
15	4.0	1.116	1.144	1.176	1.226	1.312
20	4.0	1.098	1.120	1.147	1.185	1.261

Table 23. Critical Values for Z* test statistic: Samplesize 20, shape parameter K, α levels are 0.20 thru 0.01

N	shape	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
20	6.0	1.099	1.122	1.150	1.190	1.270
20	8.0	1.100	1.123	1.151	1.193	1.269

Appendix B. Power Study Results for the Z^* Test Statistic

Power results for shape parameter k = 0.5

Table 24. Power of the Test: Samplesize N= 5, shape parameter K=0.5, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.189	0.138	0.092	0.050	0.011
2	0.755	0.673	0.553	0.384	0.123
3	0.825	0.761	0.646	0.491	0.191
4	0.508	0.421	0.303	0.183	0.043
5	0.675	0.578	0.451	0.287	0.081
6	0.768	0.679	0.563	0.419	0.148
7	0.826	0.759	0.652	0.497	0.190
8	0.782	0.701	0.573	0.412	0.137
9	0.802	0.726	0.617	0.456	0.170
10	0.770	0.693	0.570	0.412	0.155

Table 25. Power of the Test: Samplesize N=10, shape parameter K=0.5, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.199	0.152	0.105	0.053	0.011
2	0.986	0.977	0.951	0.884	0.618
3	0.997	0.992	0.984	0.948	0.771
4	0.876	0.822	0.724	0.549	0.218
5	0.968	0.943	0.897	0.787	0.449
6	0.989	0.978	0.952	0.874	0.569
7	0.997	0.993	0.981	0.950	0.782
8	0.992	0.985	0.965	0.903	0.641
9	0.996	0.991	0.977	0.929	0.712
10	0.986	0.970	0.944	0.865	0.576

Table 26. Power of the Test: Samplesize N=15, shape parameter K=0.5, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.195	0.148	0.097	0.050	0.010
2	1.000	0.999	0.997	0.987	0.905
3	1.000	1.000	1.000	0.996	0.963
4	0.969	0.947	0.900	0.777	0.435
5	0.997	0.994	0.986	0.957	0.782
6	0.999	0.998	0.994	0.976	0.841
7	1.000	1.000	1.000	0.997	0.966
8	1.000	0.999	0.997	0.990	0.913
9	1.000	1.000	0.999	0.993	0.940
10	0.999	0.998	0.994	0.976	0.842

Table 27. Power of the Test: Samplesize N=20, shape parameter K=0.5, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.199	0.148	0.101	0.052	0.010
2	1.000	1.000	1.000	0.999	0.990
3	1.000	1.000	1.000	1.000	0.998
4	0.994	0.989	0.972	0.919	0.687
5	1.000	1.000	0.998	0.992	0.949
6	1.000	1.000	0.999	0.997	0.961
7	1.000	1.000	1.000	1.000	0.997
8	1.000	1.000	1.000	1.000	0.990
9	1.000	1.000	1.000	0.999	0.993
10	1.000	1.000	0.999	0.997	0.967

Table 28. Power of the Test: Samplesize N=25, shape parameter K=0.5, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.194	0.143	0.102	0.054	0.010
2	1.000	1.000	1.000	1.000	0.998
3	1.000	1.000	1.000	1.000	1.000
4	0.999	0.996	0.994	0.976	0.840
5	1.000	1.000	1.000	1.000	0.986
6	1.000	1.000	1.000	0.999	0.994
7	1.000	1.000	1.000	1.000	1.000
8	1.000	1.000	1.000	1.000	0.999
9	1.000	1.000	1.000	1.000	1.000
10	1.000	1.000	1.000	1.000	0.991

Table 29. Power of the Test: Samplesize N=30, shape parameter K=0.5, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.207	0.152	0.102	0.054	0.013
2	1.000	1.000	1.000	1.000	1.000
3	1.000	1.000	1.000	1.000	1.000
4	1.000	0.999	0.999	0.995	0.946
5	1.000	1.000	1.000	1.000	0.999
6	1.000	1.000	1.000	1.000	0.999
7	1.000	1.000	1.000	1.000	1.000
8	1.000	1.000	1.000	1.000	1.000
9	1.000	1.000	1.000	1.000	1.000
10	1.000	1.000	1.000	1.000	0.999

Table 30. Power of the Test: Samplesize N=35, shape parameter K=0.5, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.193	0.148	0.102	0.052	0.011
2	1.000	1.000	1.000	1.000	1.000
3	1.000	1.000	1.000	1.000	1.000
4	1.000	1.000	1.000	0.997	0.973
5	1.000	1.000	1.000	1.000	1.000
6	1.000	1.000	1.000	1.000	1.000
7	1.000	1.000	1.000	1.000	1.000
8	1.000	1.000	1.000	1.000	1.000
9	1.000	1.000	1.000	1.000	1.000
10	1.000	1.000	1.000	1.000	1.000

Power results for shape parameter k = 1.0

Table 31. Power of the Test: Samplesize N= 5, shape parameter K=1.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.195	0.144	0.091	0.047	0.011
2	0.432	0.344	0.250	0.137	0.033
3	0.553	0.458	0.358	0.226	0.056
4	0.189	0.140	0.094	0.049	0.010
5	0.306	0.236	0.169	0.088	0.021
6	0.492	0.410	0.311	0.188	0.054
7	0.547	0.459	0.361	0.218	0.062
8	0.464	0.373	0.277	0.161	0.038
9	0.517	0.427	0.331	0.200	0.051
10	0.502	0.407	0.307	0.186	0.055

Table 32. Power of the Test: Samplesize N=10, shape parameter K=1.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.202	0.150	0.101	0.051	0.011
2	0.699	0.623	0.512	0.364	0.129
3	0.874	0.827	0.750	0.608	0.324
4	0.202	0.158	0.104	0.051	0.012
5	0.458	0.373	0.274	0.165	0.043
6	0.786	0.716	0.612	0.451	0.189
7	0.877	0.837	0.760	0.628	0.356
8	0.764	0.688	0.580	0.411	0.166
9	0.857	0.803	0.716	0.559	0.266
10	0.787	0.716	0.614	0.450	0.193

Table 33. Power of the Test: Samplesize N=15, shape parameter K=1.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.203	0.144	0.094	0.048	0.009
2	0.853	0.794	0.717	0.581	0.307
3	0.970	0.952	0.922	0.852	0.640
4	0.205	0.154	0.101	0.051	0.007
5	0.565	0.477	0.372	0.240	0.066
6	0.901	0.859	0.787	0.650	0.359
7	0.966	0.947	0.922	0.860	0.658
8	0.913	0.871	0.801	0.672	0.375
9	0.958	0.931	0.885	0.791	0.528
10	0.903	0.854	0.782	0.645	0.352

Table 34. Power of the Test: Samplesize N=20, shape parameter K=1.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.198	0.150	0.096	0.051	0.011
2	0.940	0.912	0.865	0.763	0.502
3	0.993	0.987	0.976	0.946	0.834
4	0.196	0.154	0.103	0.055	0.012
5	0.661	0.593	0.485	0.336	0.129
6	0.958	0.934	0.889	0.794	0.535
7	0.991	0.986	0.978	0.951	0.861
8	0.969	0.955	0.914	0.834	0.588
9	0.990	0.982	0.964	0.922	0.759
10	0.958	0.937	0.893	0.806	0.545

Table 35. Power of the Test: Samplesize N=25, shape parameter K=1.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.202	0.152	0.099	0.052	0.010
2	0.978	0.968	0.947	0.877	0.659
3	0.999	0.997	0.995	0.987	0.933
4	0.215	0.164	0.111	0.052	0.010
5	0.752	0.685	0.597	0.435	0.190
6	0.984	0.975	0.955	0.892	0.684
7	0.998	0.996	0.994	0.984	0.941
8	0.988	0.981	0.967	0.926	0.748
9	0.998	0.996	0.991	0.974	0.889
10	0.983	0.974	0.950	0.894	0.677

Table 36. Power of the Test: Samplesize N=30, shape parameter K=1.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.206	0.151	0.103	0.050	0.008
2	0.990	0.985	0.972	0.937	0.767
3	1.000	1.000	1.000	0.998	0.973
4	0.205	0.156	0.113	0.054	0.009
5	0.813	0.756	0.678	0.533	0.235
6	0.995	0.988	0.981	0.949	0.787
7	1.000	0.999	0.999	0.997	0.977
8	0.996	0.992	0.986	0.970	0.846
9	1.000	0.999	0.997	0.992	0.947
10	0.995	0.989	0.981	0.952	0.779

Table 37. Power of the Test: Samplesize N=35, shape parameter K=1.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.197	0.150	0.097	0.049	0.010
2	0.994	0.991	0.983	0.961	0.858
3	1.000	1.000	1.000	0.998	0.990
4	0.195	0.151	0.102	0.050	0.008
5	0.851	0.806	0.725	0.577	0.296
6	0.997	0.994	0.987	0.965	0.842
7	1.000	1.000	0.999	0.998	0.992
8	0.999	0.997	0.995	0.986	0.919
9	1.000	1.000	1.000	0.998	0.985
10	0.997	0.995	0.991	0.970	0.862

Power results for shape parameter k = 1.5

Table 38. Power of the Test: Samplesize N= 5, shape parameter K=1.5, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.188	0.138	0.087	0.045	0.011
2	0.265	0.197	0.129	0.071	0.014
3	0.380	0.304	0.215	0.122	0.024
4	0.100	0.071	0.045	0.022	0.004
5	0.173	0.130	0.088	0.044	0.009
6	0.341	0.275	0.195	0.112	0.026
7	0.372	0.302	0.214	0.122	0.028
8	0.297	0.229	0.158	0.080	0.018
9	0.351	0.281	0.196	0.113	0.025
10	0.339	0.271	0.186	0.111	0.026

Table 39. Power of the Test: Samplesize N=10, shape parameter K=1.5, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.203	0.147	0.100	0.050	0.012
2	0.362	0.281	0.198	0.114	0.025
3	0.630	0.542	0.444	0.310	0.107
4	0.058	0.043	0.025	0.012	0.001
5	0.164	0.118	0.077	0.040	0.007
6	0.536	0.440	0.350	0.225	0.066
7	0.638	0.557	0.463	0.333	0.134
8	0.435	0.343	0.260	0.156	0.042
9	0.606	0.504	0.405	0.276	0.078
10	0.532	0.445	0.351	0.226	0.071

Table 40. Power of the Test: Samplesize N=15, shape parameter K=1.5, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.197	0.147	0.090	0.047	0.009
2	0.442	0.365	0.274	0.163	0.041
3	0.786	0.725	0.638	0.503	0.248
4	0.026	0.019	0.010	0.003	0.001
5	0.130	0.094	0.055	0.026	0.005
6	0.636	0.565	0.458	0.314	0.109
7	0.789	0.729	0.647	0.525	0.272
8	0.570	0.487	0.378	0.239	0.072
9	0.731	0.663	0.564	0.408	0.163
10	0.626	0.546	0.450	0.305	0.117

Table 41. Power of the Test: Samplesize N=20, shape parameter K=1.5, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.197	0.144	0.097	0.051	0.010
2	0.520	0.435	0.343	0.216	0.066
3	0.868	0.826	0.763	0.647	0.410
4	0.019	0.012	0.007	0.002	0.000
5	0.123	0.093	0.059	0.026	0.006
6	0.724	0.653	0.558	0.419	0.175
7	0.879	0.841	0.782	0.684	0.457
8	0.656	0.577	0.470	0.322	0.114
9	0.843	0.789	0.710	0.562	0.276
10	0.738	0.659	0.569	0.418	0.170

Table 42. Power of the Test: Samplesize N=25, shape parameter K=1.5, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.207	0.159	0.106	0.051	0.010
2	0.591	0.509	0.417	0.260	0.093
3	0.933	0.906	0.864	0.781	0.546
4	0.012	0.007	0.005	0.001	0.000
5	0.133	0.097	0.066	0.031	0.005
6	0.803	0.743	0.658	0.512	0.242
7	0.935	0.913	0.875	0.794	0.587
8	0.747	0.676	0.575	0.411	0.165
9	0.916	0.884	0.826	0.707	0.406
10	0.800	0.743	0.655	0.507	0.241

Table 43. Power of the Test: Samplesize N=30, shape parameter K=1.5, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.198	0.147	0.099	0.046	0.008
2	0.629	0.548	0.446	0.317	0.116
3	0.958	0.939	0.909	0.854	0.667
4	0.005	0.003	0.002	0.001	0.000
5	0.112	0.081	0.050	0.025	0.005
6	0.851	0.801	0.716	0.586	0.304
7	0.959	0.941	0.911	0.855	0.677
8	0.800	0.725	0.637	0.481	0.213
9	0.947	0.915	0.866	0.769	0.507
10	0.851	0.787	0.700	0.572	0.305

Table 44. Power of the Test: Samplesize N=35, shape parameter K=1.5, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.205	0.151	0.100	0.050	0.010
2	0.681	0.600	0.500	0.356	0.137
3	0.976	0.965	0.943	0.900	0.755
4	0.004	0.001	0.001	0.000	0.000
5	0.111	0.077	0.049	0.024	0.005
6	0.867	0.821	0.745	0.621	0.350
7	0.976	0.965	0.945	0.907	0.779
8	0.845	0.784	0.695	0.557	0.262
9	0.975	0.960	0.927	0.848	0.602
10	0.886	0.838	0.760	0.639	0.355

Power results for shape parameter k = 2.0

Table 45. Power of the Test: Samplesize N= 5, shape parameter K=2.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.188	0.142	0.088	0.042	0.012
2	0.188	0.137	0.089	0.046	0.010
3	0.291	0.226	0.153	0.082	0.017
4	0.068	0.049	0.030	0.014	0.002
5	0.124	0.095	0.060	0.029	0.006
6	0.274	0.215	0.142	0.076	0.018
7	0.290	0.229	0.156	0.084	0.017
8	0.220	0.165	0.108	0.054	0.013
9	0.275	0.209	0.145	0.075	0.015
10	0.272	0.206	0.139	0.079	0.017

Table 46. Power of the Test: Samplesize N=10, shape parameter K=2.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.206	0.146	0.098	0.051	0.012
2	0.189	0.136	0.089	0.046	0.008
3	0.431	0.349	0.267	0.166	0.047
4	0.028	0.018	0.010	0.004	0.000
5	0.078	0.050	0.031	0.014	0.003
6	0.376	0.294	0.219	0.128	0.035
7	0.445	0.368	0.290	0.193	0.060
8	0.259	0.189	0.131	0.071	0.015
9	0.406	0.327	0.238	0.134	0.029
10	0.378	0.300	0.218	0.134	0.034

Table 47. Power of the Test: Samplesize N=15, shape parameter K=2.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.205	0.148	0.088	0.046	0.008
2	0.202	0.148	0.094	0.045	0.008
3	0.557	0.476	0.372	0.254	0.089
4	0.008	0.005	0.002	0.001	0.000
5	0.037	0.025	0.014	0.007	0.001
6	0.438	0.357	0.260	0.156	0.044
7	0.564	0.492	0.394	0.272	0.109
8	0.305	0.234	0.158	0.086	0.020
9	0.497	0.412	0.304	0.194	0.046
10	0.427	0.342	0.259	0.164	0.048

Table 48. Power of the Test: Samplesize N=20, shape parameter K=2.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.199	0.144	0.094	0.050	0.009
2	0.197	0.147	0.096	0.051	0.011
3	0.630	0.558	0.474	0.338	0.140
4	0.003	0.002	0.000	0.000	0.000
5	0.026	0.019	0.011	0.005	0.000
6	0.491	0.407	0.313	0.197	0.064
7	0.650	0.590	0.512	0.383	0.178
8	0.320	0.249	0.174	0.092	0.022
9	0.578	0.492	0.378	0.249	0.078
10	0.494	0.411	0.315	0.193	0.061

Power results for shape parameter k = 3.0

Table 49. Power of the Test: Samplesize N= 5, shape parameter K=3.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.194	0.141	0.090	0.040	0.011
2	0.130	0.093	0.059	0.027	0.007
3	0.210	0.157	0.104	0.048	0.011
4	0.048	0.031	0.019	0.008	0.001
5	0.089	0.063	0.039	0.018	0.003
6	0.210	0.153	0.096	0.050	0.012
7	0.217	0.162	0.106	0.050	0.010
8	0.156	0.113	0.071	0.032	0.007
9	0.202	0.146	0.096	0.047	0.009
10	0.204	0.147	0.102	0.049	0.011

Table 50. Power of the Test: Samplesize N=10, shape parameter K=3.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.198	0.148	0.102	0.055	0.012
2	0.079	0.054	0.032	0.014	0.002
3	0.242	0.185	0.128	0.065	0.014
4	0.010	0.006	0.003	0.001	0.000
5	0.030	0.018	0.012	0.005	0.001
6	0.224	0.172	0.119	0.062	0.013
7	0.254	0.196	0.142	0.081	0.016
8	0.121	0.083	0.051	0.026	0.004
9	0.220	0.161	0.111	0.052	0.008
10	0.225	0.167	0.121	0.069	0.013

Table 51. Power of the Test: Samplesize N=15, shape parameter K=3.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.204	0.153	0.091	0.044	0.007
2	0.057	0.037	0.021	0.009	0.001
3	0.266	0.206	0.141	0.075	0.018
4	0.001	0.001	0.001	0.000	0.000
5	0.010	0.006	0.003	0.001	0.000
6	0.234	0.171	0.109	0.056	0.011
7	0.279	0.221	0.154	0.088	0.028
8	0.107	0.072	0.040	0.019	0.003
9	0.230	0.171	0.108	0.052	0.009
10	0.230	0.173	0.121	0.062	0.015

Table 52. Power of the Test: Samplesize N=20, shape parameter K=3.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.198	0.143	0.095	0.048	0.009
2	0.045	0.029	0.018	0.007	0.001
3	0.284	0.221	0.160	0.091	0.020
4	0.000	0.000	0.000	0.000	0.000
5	0.005	0.003	0.001	0.000	0.000
6	0.240	0.173	0.116	0.066	0.014
7	0.319	0.249	0.179	0.111	0.033
8	0.083	0.056	0.036	0.014	0.001
9	0.233	0.169	0.115	0.057	0.008
10	0.229	0.168	0.117	0.065	0.016

Power results for shape parameter k = 4.0

Table 53. Power of the Test: Samplesize N= 5, shape parameter K=4.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.194	0.141	0.089	0.039	0.011
2	0.107	0.075	0.046	0.021	0.005
3	0.178	0.127	0.083	0.036	0.008
4	0.037	0.025	0.014	0.007	0.000
5	0.071	0 051	0.032	0.013	0.002
6	0.178	0.122	0.080	0.040	0.009
7	0.182	0.131	0.084	0.038	0.007
8	0.128	0.089	0.055	0.025	0.005
9	0.169	0.121	0.078	0.034	0.006
10	0.174	0.123	0.082	0.037	0.007

Table 54. Power of the Test: Samplesize N=10, shape parameter K=4.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.201	0.150	0.100	0.055	0.011
2	0.048	0.031	0.017	0.007	0.000
3	0.167	0.118	0.072	0.041	0.006
4	0.006	0.004	0.001	0.000	0.000
5	0.017	0.011	0.006	0.003	0.000
6	0.168	0.123	0.079	0.039	0.007
7	0.175	0.129	0.086	0.046	0.009
8	0.073	0.047	0.032	0.015	0.001
9	0.147	0.104	0.062	0.028	0.004
10	0.163	0.124	0.084	0.041	0.007

Table 55. Power of the Test: Samplesize N=15, shape parameter K=4.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.207	0.149	0.096	0.044	0.008
2	0.028	0.017	0.010	0.003	0.001
3	0.157	0.112	0.073	0.035	0.007
4	0.001	0.000	0.000	0.000	0.000
5	0.005	0.003	0.001	0.000	0.000
6	0.150	0.105	0.064	0.029	0.005
7	0.168	0.126	0.083	0.043	0.011
8	0.052	0.034	0.020	0.009	0.001
9	0.131	0.090	0.054	0.022	0.003
10	0.155	0.115	0.074	0.032	0.007

Table 56. Power of the Test: Samplesize N=20, shape parameter K=4.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.196	0.146	0.096	0.051	0.008
2	0.018	0.013	0.006	0.003	0.000
3	0.150	0.112	0.069	0.032	0.005
4	0.000	0.000	0.000	0.000	0.000
5	0.001	0.001	0.000	0.000	0.000
6	0.141	0.099	0.068	0.031	0.004
7	0.165	0.123	0.084	0.045	0.012
8	0.036	0.025	0.012	0.004	0.000
9	0.123	0.086	0.048	0.019	0.002
10	0.138	0.102	0.063	0.032	0.006

Power results for shape parameter k = 6.0 and 8.0

Table 57. Power of the Test: Samplesize N=20, shape parameter K=6.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.196	0.142	0.092	0.049	0.009
2	0.007	0.004	0.002	0.001	0.000
3	0.066	0.042	0.023	0.009	0.001
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.077	0.053	0.029	0.010	0.001
7	0.077	0.051	0.033	0.015	0.002
8	0.013	0.008	0.003	0.000	0.000
9	0.051	0.030	0.015	0.004	0.000
10	0.079	0.053	0.030	0.014	0.001

Table 58. Power of the Test: Samplesize N=20, shape parameter K=8.0, α levels are 0.20 thru 0.01

Distribution	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.196	0.141	0.092	0.049	0.010
2	0.004	0.003	0.001	0.000	0.000
3	0.039	0.024	0.012	0.004	0.000
4	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000
6	0.057	0.037	0.019	0.005	0.000
7	0.047	0.032	0.018	0.009	0.001
8	0.008	0.003	0.001	0.000	0.000
9	0.031	0.016	0.009	0.002	0.000
10	0.055	0.037	0.020	0.008	0.001

Appendix C. Graphical Representations of H_0 versus H_a

Table 59. Parametric Distribution Functions Utilized throughout this Thesis

Distribution	Identification:Parameters
1a	Weibull with shape parameter = 0.5
1b	Weibull with shape parameter = 1.0
1c	Weibull with shape parameter = 1.5
2	Weibull with shape parameter = 2.0
3	Weibull with shape parameter = 3.5
4	Gamma with shape parameter = 1.0
5	Gamma with shape parameter = 2.0
6	Uniform with a= 0.0 and b= 1.0
7	Normal with mean= 0.0 and stdev= 1.0
8	Beta with P=2.0 and Q=3.0
9	Beta with P=2.0 and Q=2.0
10	Beta with P=1.0 and Q=1.0

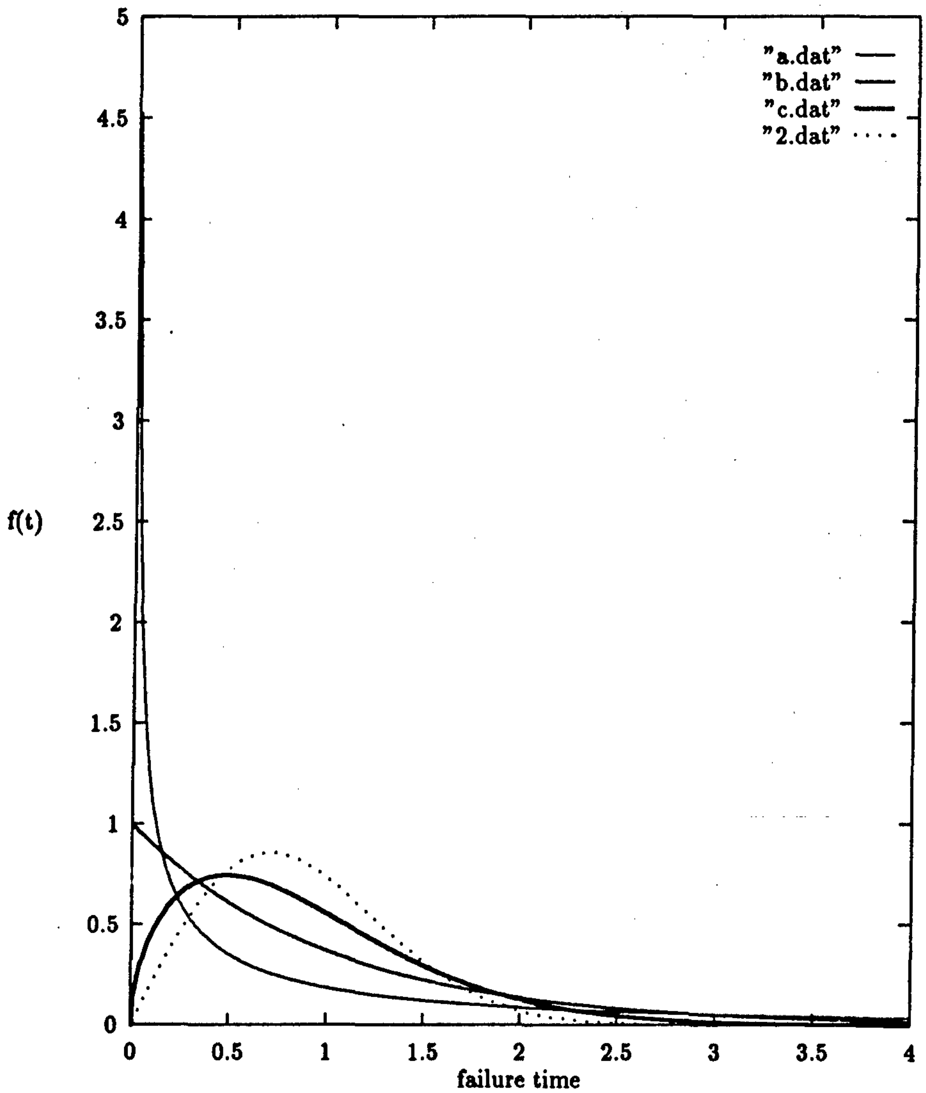

Figure 10. Graph of Weibull distributions: k = 0.5, 1.0, 1.5, 2.0

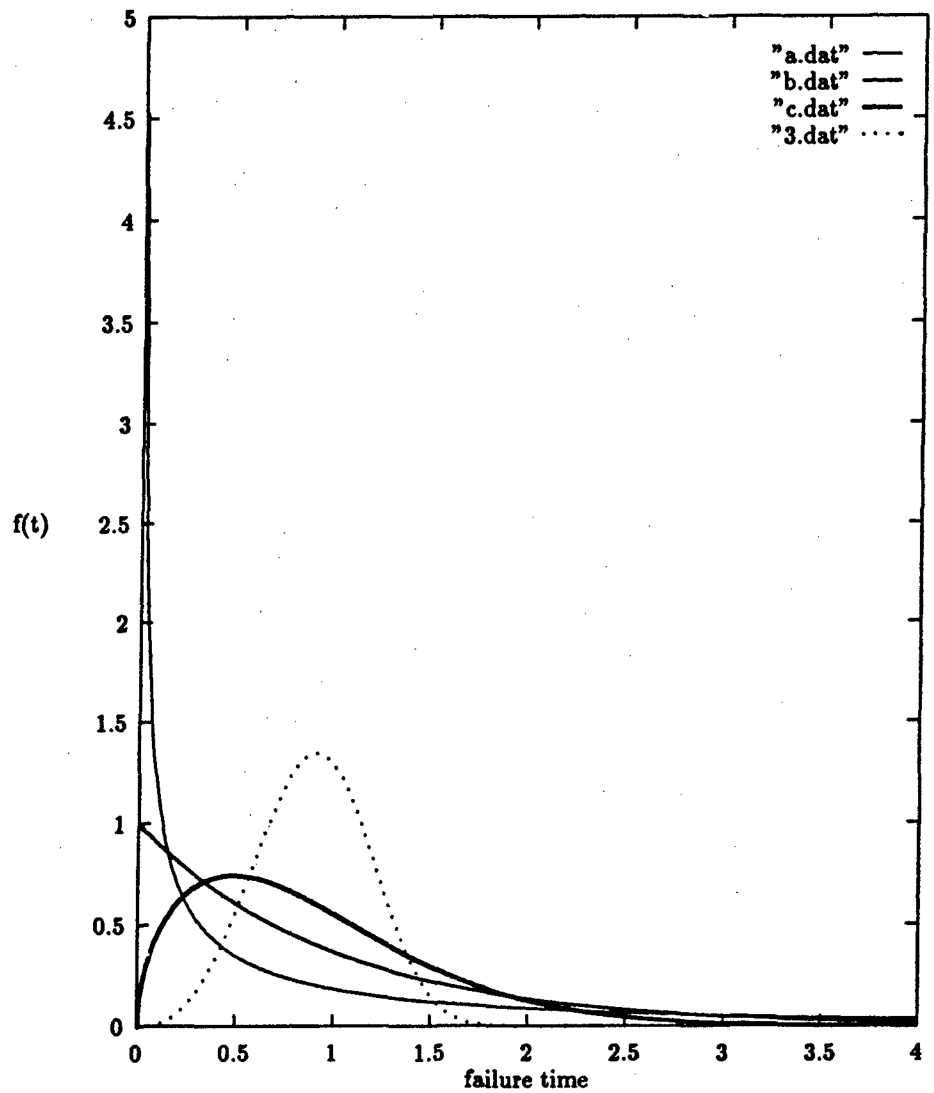

Figure 11. Graph of Weibull distributions: k = 0.5, 1.0, 1.5, 3.5

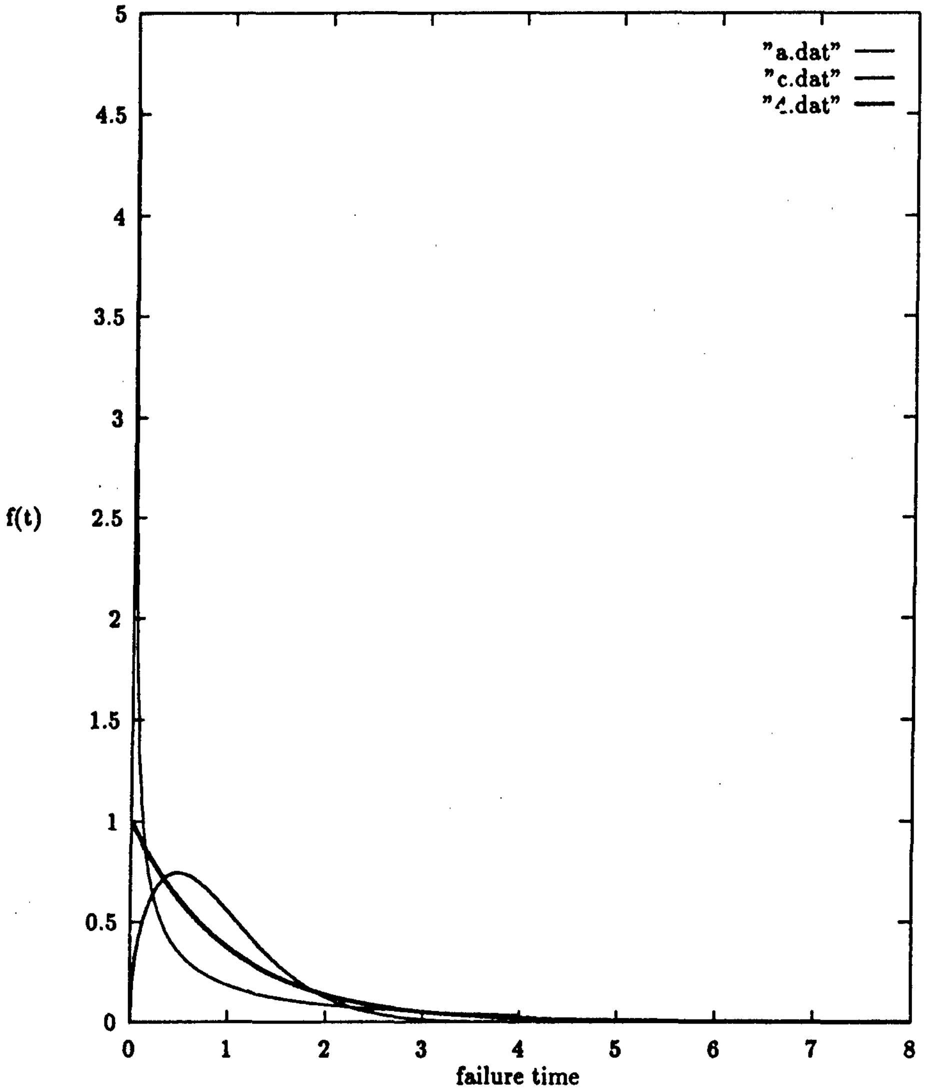

Figure 12. Graph of Weibull distributions; k = 0.5 and k = 1.5 and of the Gamma Distribution: shape parameter k = 1.0

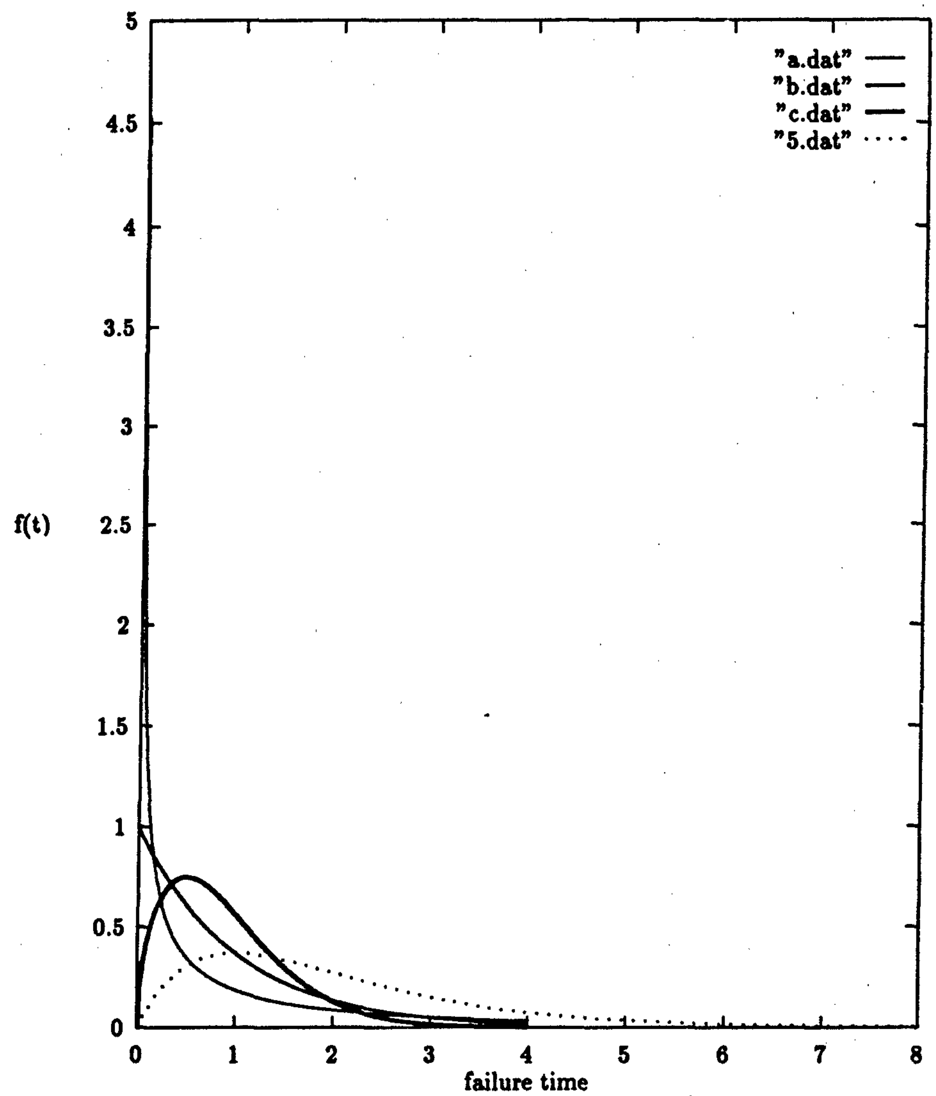

Figure 13. Graph of Weibull distributions: k = 0.5, 1.0, 1.5 and of the Gamma Distribution with shape parameter k = 2.0

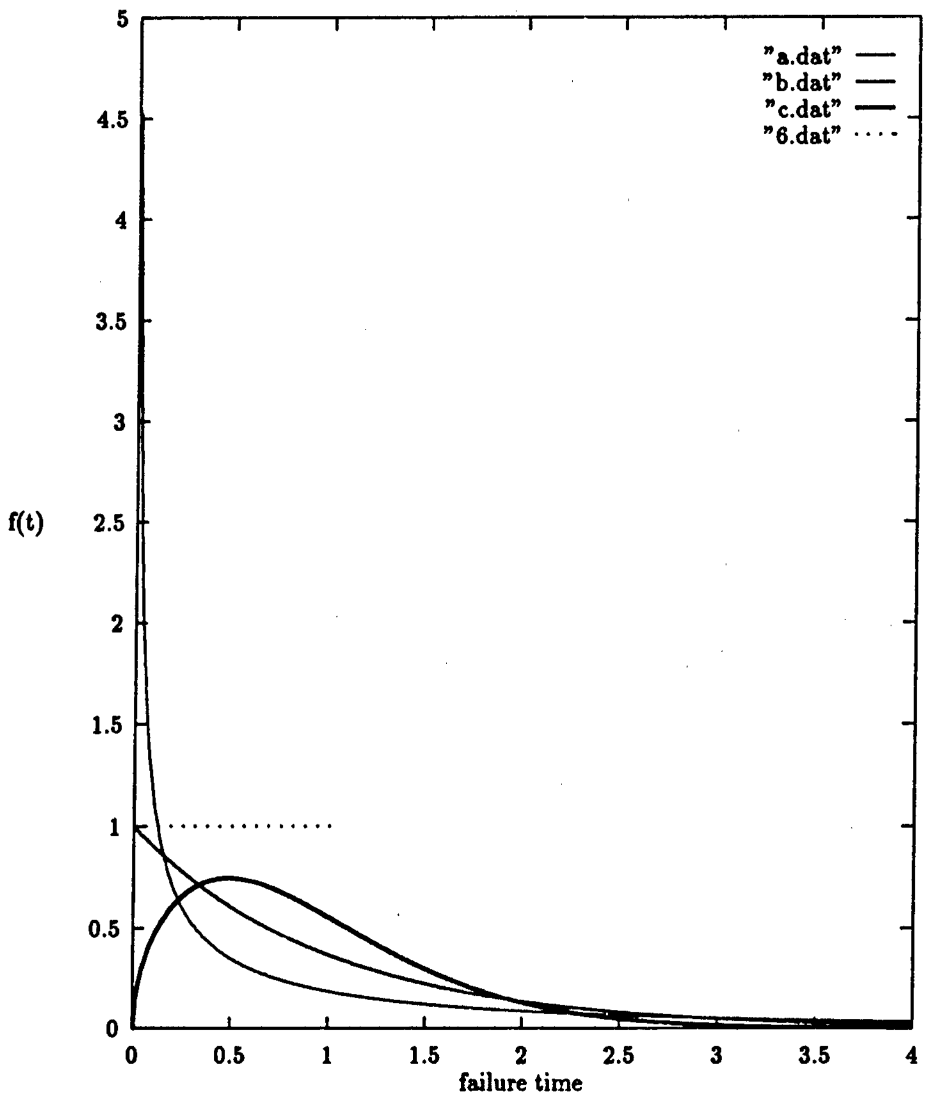

Figure 14. Graph of Weibull distributions: k = 0.5, 1.0, 1.5 and of the Uniform Distribution with parameters (0,1)

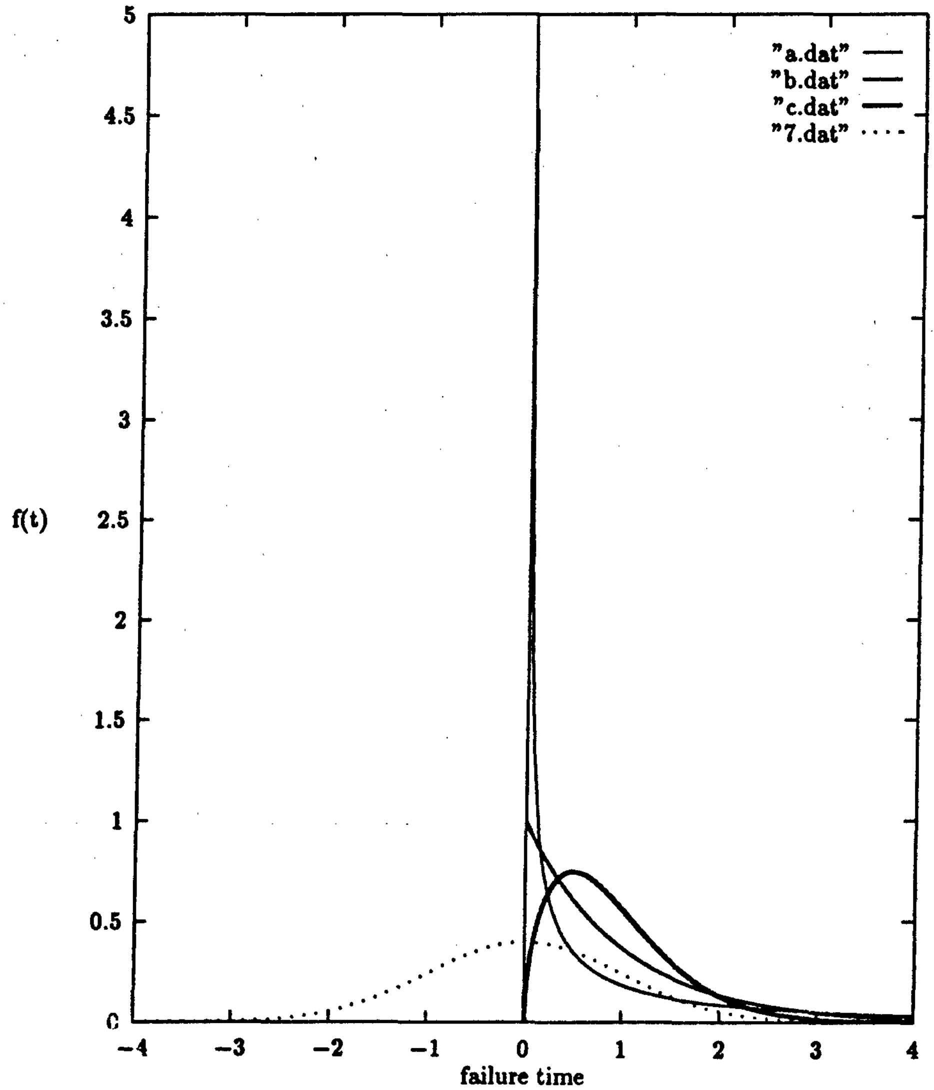

Figure 15. Graph of Weibull distributions: k = 0.5, 1.0, 1.5 and of the Normal Distribution with mean = 0 and std.dev = 1.0

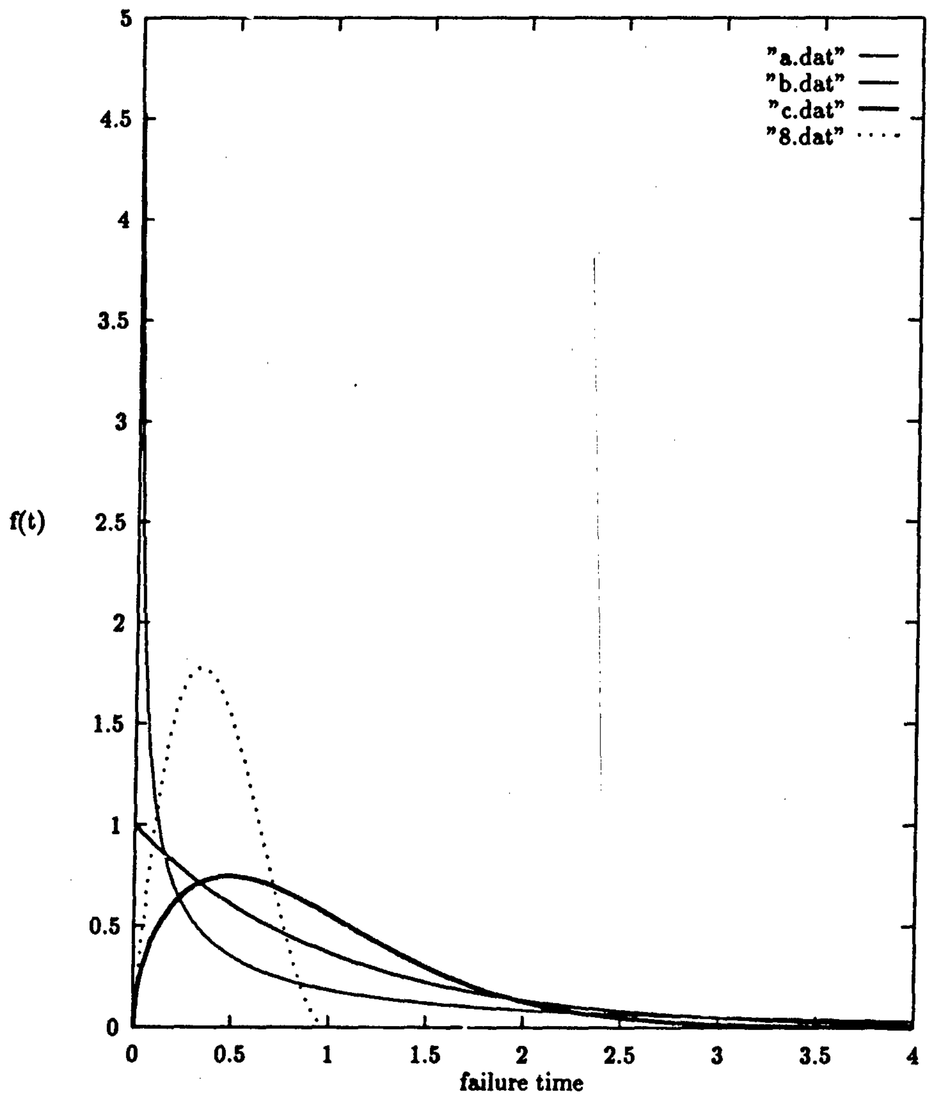

Figure 16. Graph of Weibull distributions: k = 0.5, 1.0, 1.5 and of the Beta Distribution with p = 2.0 and q = 3.0

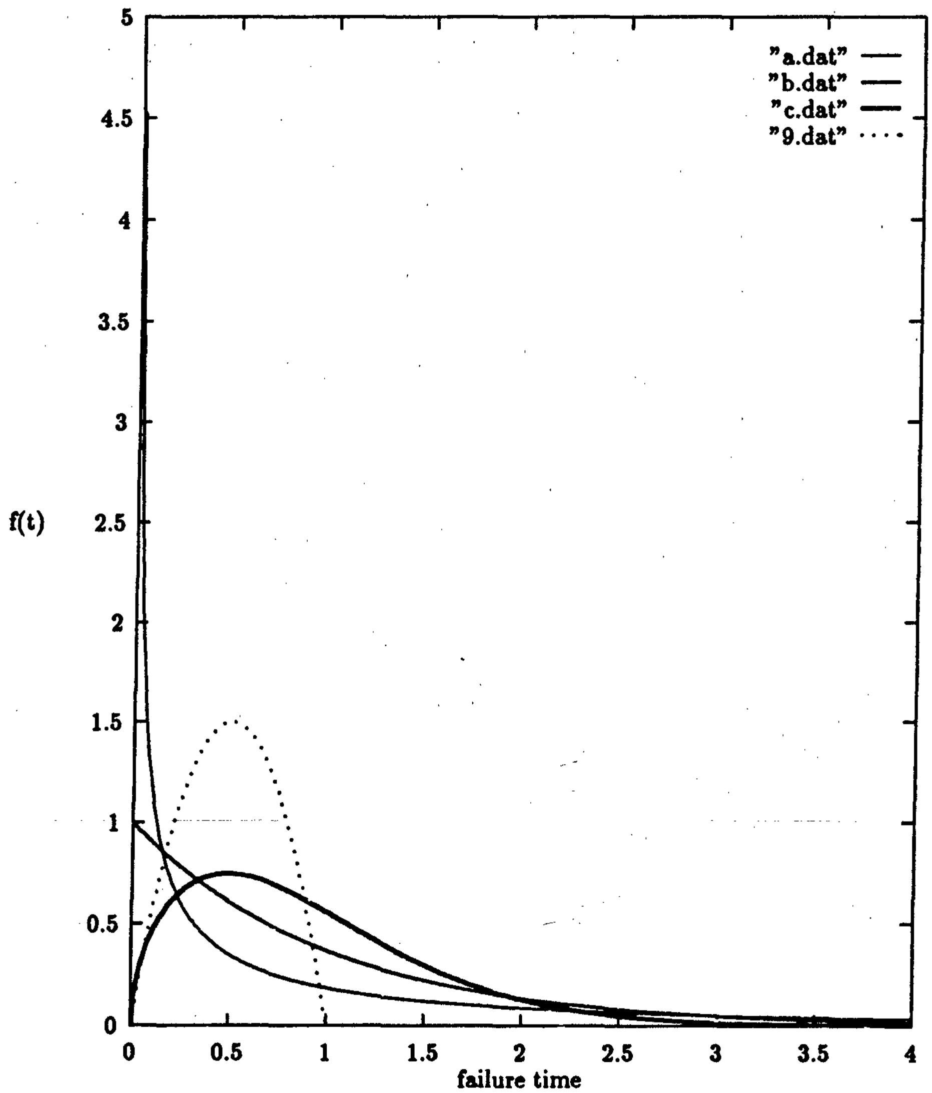

Figure 17. Graph of Weibull distributions: k = 0.5, 1.0, 1.5 and of the Beta Distribution with p = 2.0 and q = 2.0

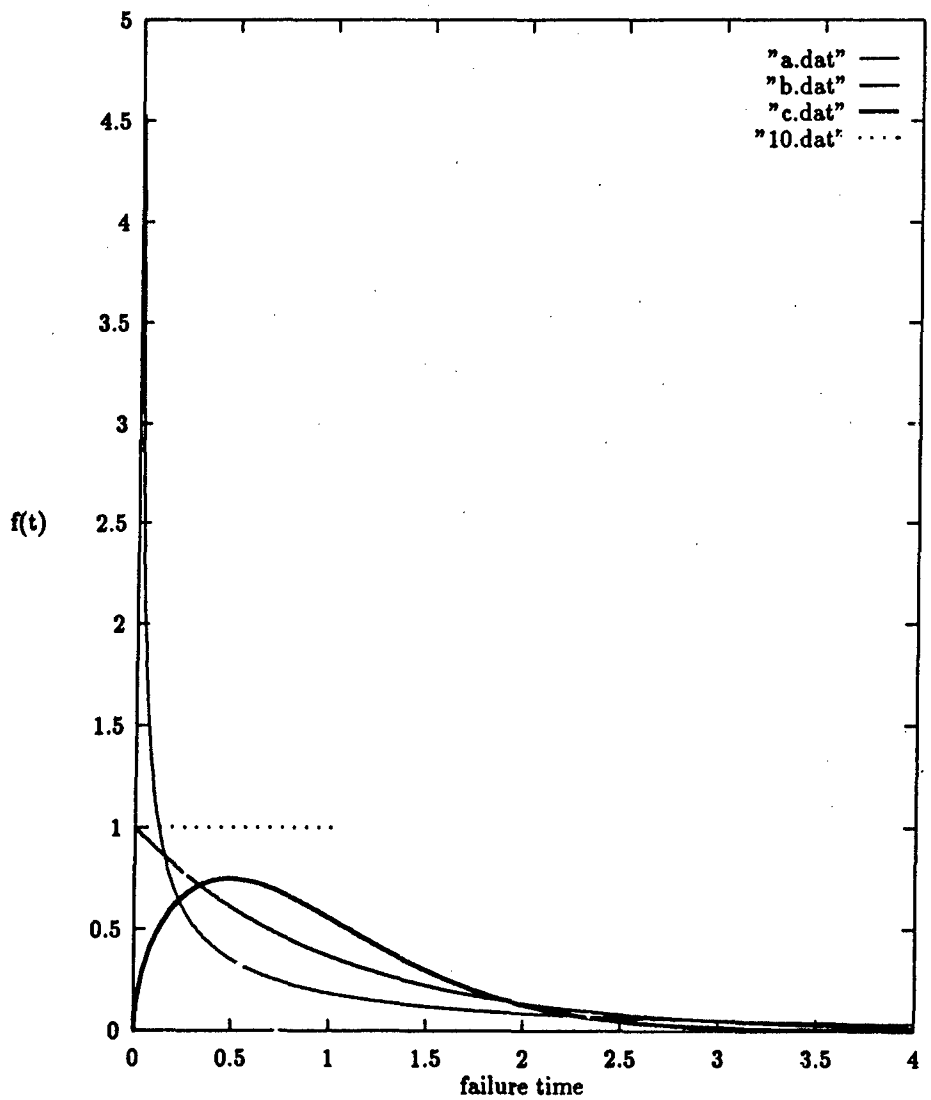

Figure 18. Graph of Weibull distributions: k = 0.5, 1.0, 1.5 and of the Beta Distribution with p = 1.0 and q = 1.0

Appendix D. Expected Value Differences for the Standard Weibull Distribution

Table 60. Calculated MuDif Values for use in the denominator of the Z^* Statistic: Sample size 5, shape parameter k=0.5 ,1.0, 1.5

MuDif #	k=0.5	k=1.0	k=1.5
1	0.225	0.25	0.2476
2	0.5222	0.3333	0.2622
3	1.283	0.5	0.3274
4	4.567	1.0	0.5383

Table 61. Calculated MuDif Values for use in the denominator of the Z^* Statistic: Sample size 10, shape parameter k=0.5,1.0, 1.5

MuDif #	k=0.5	k=1.0	k=1.5
1	0.04691	0.1111	0.1415
2	0.08403	0.125	0.1301
3	0.1368	0.1429	0.1293
4	0.2152	0.1667	0.1349
5	0.3383	0.2	0.1469
6	0.5478	0.25	0.1676
7	0.9526	0.3333	0.2039
8	1.929	0.5	0.2766
9	5.858	1.0	0.4839

Table 62. Calculated MuDif Values for use in the denominator of the Z^* Statistic: Sample size 15, shape parameter k=0.5,1.0, 1.5

MuDif #	k=0.5	k=1.0	k=1.5
1	0.01973	0.07143	0.1048
2	0.03308	0.07692	0.09289
3	0.04973	0.08333	0.08828
4	0.07077	0.09091	0.08704
5	0.09785	0.1	0.08801
6	0.1334	0.1111	0.09083
7	0.1813	0.125	0.09556
8	0.2481	0.1429	0.1026
9	0.345	0.1667	0.1126
10	0.494	0.2	0.1272
11	0.7424	0.25	0.1494
12	1.212	0.3333	0.1861
13	2.318	0.5	0.2578
14	6.636	1.0	0.4604

Table 63. Calculated MuDif Values for use in the denominator of the Z^* Statistic: Sample size 20, shape parameter k=0.5,1.0, 1.5

MuDif #	k=0.5	k=1.0	k=1.5
1	0.0108	0.05263	0.08524
2	0.01758	0.05556	0.07431
3	0.02553	0.05882	0.06928
4	0.03494	0.0625	0.06681
5	0.04616	0.06667	0.06582
6	0.05966	0.07143	0.06587
7	0.07608	0.07692	0.06678
8	0.09631	0.08333	0.06846
9	0.1216	0.09091	0.07096
10	0.1538	0.1	0.07437
11	0.1955	0.1111	0.07887
12	0.2512	0.125	0.08478
13	0.3279	0.1429	0.0926
14	0.4381	0.1667	0.1032
15	0.6058	0.2	0.1181
16	0.8822	0.25	0.1402
17	1.398	0.3333	0.1766
18	2.598	0.5	0.2471
19	7.195	1.0	0.4462

Table 64. Calculated MuDif Values for use in the denominator of the Z^* Statistic: Sample size 25, shape parameter k=0.5,1.0, 1.5

MuDif #	k=0.5	k=1.0	k=1.5
1	0.006806	0.04167	0.07282
2	0.01088	0.04348	0.06288
3	0.01551	0.04545	0.058
4	0.02078	0.04762	0.05528
5	0.02682	0.05	0.05374
6	0.03377	0.05263	0.05299
7	0.04182	0.05556	0.05281
8	0.0512	0.05882	0.0531
9	0.06222	0.0625	0.05381
10	0.07525	0.06667	0.05493
11	0.09083	0.07143	0.05647
12	0.1097	0.07692	0.05846
13	0.1327	0.08333	0.06097
14	0.1613	0.09091	0.0641
15	0.1974	0.1	0.06798
16	0.244	0.1111	0.07285
17	0.3058	0.125	0.07902
18	0.3903	0.1429	0.08702
19	0.5109	0.1667	0.09768
20	0.6931	0.2	0.1125
21	0.9913	0.25	0.1345
22	1.544	0.3333	0.1704
23	2.816	0.5	0.2399
24	7.632	1.0	0.4363

Table 65. Calculated MuDif Values for use in the denominator of the Z^* Statistic: Sample size 30, shape parameter k=0.5,1.0, 1.5

MuDif #	k=0.5	k=1.0	k=1.5
1	0.004677	0.03448	0.06412
2	0.007395	0.03571	0.05502
3	0.01041	0.03704	0.05041
4	0.01377	0.03846	0.04769
5	0.01752	0.04	0.046
6	0.02172	0.04167	0.04495
7	0.02645	0.04348	0.04437
8	0.03178	0.04545	0.04413
9	0.03783	0.04762	0.0442
10	0.04472	0.05	0.04451
11	0.05262	0.05263	0.04507
12	0.06172	0.05555	0.04587
13	0.07227	0.05882	0.04691
14	0.08459	0.0625	0.04821
15	0.09912	0.06667	0.04978
16	0.1164	0.07142	0.05173
17	0.1372	0.07693	0.05396
18	0.1625	0.08332	0.0568
19	0.1938	0.09096	0.06006
20	0.2332	0.09994	0.06408
21	0.2838	0.1112	0.06905
22	0.3505	0.125	0.07527
23	0.4414	0.1429	0.08335
24	0.5706	0.1666	0.09394
25	0.7647	0.2	0.1087
26	1.081	0.25	0.1304
27	1.663	0.3333	0.1659
28	2.995	0.5	0.2346
29	7.99	1.0	0.4289

Table 66. Calculated MuDif Values for use in the denominator of the Z^* Statistic: Sample size 35, shape parameter k=0.5, 1.0, 1.5

MuDif #	k=0.5	k=1.0	k=1.5
1	0.003411	0.02941	0.05762
2	0.005351	0.0303	0.04923
3	0.007471	0.03125	0.0449
4	0.009793	0.03226	0.04226
5	0.01234	0.03333	0.04054
6	0.01515	0.03448	0.03939
7	0.01824	0.03571	0.03863
8	0.02166	0.03704	0.03817
9	0.02545	0.03846	0.03794
10	0.02967	0.04	0.0379
11	0.03437	0.04167	0.03804
12	0.03965	0.04348	0.03832
13	0.04558	0.04546	0.03882
14	0.05229	0.0476	0.03915
15	0.0599	0.04992	0.04053
16	0.0686	0.05272	0.04022
17	0.07861	0.05548	0.04188
18	0.09007	0.05916	0.04418
19	0.1036	0.06315	0.04937
20	0.1196	0.06766	0.04211
21	0.1379	0.06578	0.05429
22	0.1603	0.08297	0.04773
23	0.1879	0.07869	0.05383
24	0.2216	0.09759	0.0514
25	0.2636	0.0911	0.07037
26	0.3172	0.1179	0.06261
27	0.3888	0.1213	0.07243
28	0.4847	0.1445	0.07733
29	0.6212	0.1663	0.09474
30	0.8254	0.2002	0.1041
31	1.157	0.2499	0.1277
32	1.765	0.3334	0.1624
33	3.147	0.5	0.2305
34	8.294	1.0	0.423

Appendix E. Fortran Program for Determining Critical Values for the Z^* Test Statistic

```
*************************************************************
THIS PROGRAM COMPUTES THE CRITICAL VALUES FOR THE Z* STATISTIC
*************************************************************
*
INTEGER S,I,J,II,JJ,SS,N,SEED,PASSNO,IERROR
*
REAL CRIT80,CRIT85,CRIT90,CRIT95,CRIT99,K
*
REAL Y(35),X(35),GAP(35),MUDIF(34)
       REAL XX(35),ZSTAR(0:10003),G(35)
REAL NUMSUM,NUM,DENSUM,DENOM
*
EXTERNAL RNSET,SVRGN,RNWIB
*
*            set shape parameter for this initial run
      K=1.0
S=10000
*
OPEN(UNIT=35,FILE='d_n35_k1.prn',STATUS='OLD',IOSTAT=
    +                IERROR, ERR=1000)
OPEN(UNIT=30,FILE='d_n30_k1.prn',STATUS='OLD',IOSTAT=
    +                IERROR, ERR=1000)
OPEN(UNIT=25,FILE='d_n25_k1.prn',STATUS='OLD',IOSTAT=
    +                IERROR, ERR=1000)
OPEN(UNIT=20,FILE='d_n20_k1.prn',STATUS='OLD',IOSTAT=
    +                IERROR, ERR=1000)
OPEN(UNIT=15,FILE='d_n15_k1.prn',STATUS='OLD',IOSTAT=
    +                IERROR, ERR=1000)
OPEN(UNIT=10,FILE='d_n10_k1.prn',STATUS='OLD',IOSTAT=
    +                IERROR, ERR=1000)
OPEN(UNIT=5,FILE='d_n5_k1.prn',STATUS='OLD',IOSTAT=
    +                IERROR, ERR=1000)
*
OPEN(UNIT=22,FILE='zcrit.out',STATUS='UNKNOWN')
       OPEN(UNIT=33,FILE='mudif.out')
       OPEN(UNIT=44,FILE='yvals.dat')
OPEN(UNIT=55,FILE='xvals.dat')
OPEN(UNIT=66,FILE='xxvals.dat')
```

```
        OPEN(UNIT=77,FILE='gaps.dat')
OPEN(UNIT=88,FILE='gi.dat')
OPEN(UNIT=99,FILE='zstar.dat')
        OPEN(UNIT=93,FILE='zsort.out')
*
DO 3000 PASSNO=1,1
IF(PASSNO.EQ.1)THEN
        SEED=402958961
ELSE
   SEED=310495
ENDIF
CALL RNSET(SEED)
*
DO 2000 N=5,35,5
*
I=0
J=0
II=0
JJ=0
SS=0
DO 5 I=1,N
X(I)=0.0
Y(I)=0.0
GAP(I)=0.0
G(I)=0.0
  5 CONTINUE
                        DO 6 I=0,S+1
ZSTAR(I)=0.0
  6 CONTINUE
*
CRIT80=0.0
CRIT85=0.0
CRIT90=0.0
CRIT95=0.0
CRIT99=0.0
*
READ(N,*)(MUDIF(I),I=1,N-1)
                        WRITE(33,903)(MUDIF(I),I=1,N-1)
 903 FORMAT('EXPECTED VALUE GAPS ',/,
    +
                   DO 110 J=1,S
*
```

```
CALL RNWIB(N,K,Y)
*
WRITE(44,904)(Y(I),I=1,N)
 904 FORMAT(1X,' RAW DATA SET IS',/,
     +                                5(4F10.6)/)
*
DO 7 I=1,N
        X(I)=Y(I)+10.0
  7 CONTINUE
WRITE(55,905)(X(I),I=1,N)
 905 FORMAT(1X,' SCALED DATA SET IS',/,
     +                                5(4F10.6)/)
*
CALL SVRGN(N,X,XX)
WRITE(66,906)(XX(I),I=1,N)
 906 FORMAT(1X,' SORTED DATA SET IS',/,
     +                                5(4F10.6)/)
*
DO 8 I=1,N-1
GAP(I)=XX(I+1)-XX(I)
  8 CONTINUE
                    WRITE(77,907)(GAP(I),I=1,N-1)
 907                FORMAT(1X,'GAPS ARE ',/,5(4F10.6),/)
*
DO 9 I=1,N-1
G(I)=GAP(I)/MUDIF(I)
  9 CONTINUE
                    WRITE(88,908)(G(I),I=1,N-1)
 908                FORMAT(1X,'G(I) VALS ',/,5(4F10.6),/)
*
NUMSUM=0.0
*
DO 10 I=1,N-2
NUMSUM=NUMSUM+(N-1-I)*G(I)
 10 CONTINUE
NUM=2.0*NUMSUM
*
DENSUM=0.0
DO 11 I=1,N-1
DENSUM=DENSUM+G(I)
 11 CONTINUE
DENOM=(N-2)*DENSUM
```

```
*
ZSTAR(J)=NUM/DENOM
*
WRITE(99,909) ZSTAR(J)
 909                  FORMAT(1X,'Z* VALUE IS ' ,F10.6)

 110    CONTINUE
*
SS=S+1
*
CALL SVRGN(SS,ZSTAR,ZSTAR)
*
CALL EXTRA(S,ZSTAR)
*
CALL VALUES(ZSTAR,CRIT80,CRIT85,CRIT90,
     +                              CRIT95,CRIT99,SS)
*
                WRITE(93,913)ZSTAR(8000),ZSTAR(8500),ZSTAR(9000),
     +                    ZSTAR(9500),ZSTAR(9900)
 913            FORMAT(1X,5F9.4)

*
WRITE(22,902)N,K,CRIT80,CRIT85,CRIT90,
     +                        CRIT95,CRIT99
*
        CLOSE(N)
 2000 CONTINUE
 3000   CONTINUE
*
STOP
*
 1000   WRITE(*,1001) IERROR
 1001   FORMAT(1X,'IOSTAT = ', IERROR/)
*
END
*****************************************************************
*                                                               *
*    USING THE TECHNIQUE EXPLAINED IN CHAPTER 3, FIND THE       *
*    CRITICAL VALUES                                            *
*                                                               *
*****************************************************************
      SUBROUTINE CAN(Y1,Y2,D1,D2,Y,RES)
```

```
      REAL M,B,Y1,Y2,D1,D2,Y,RES
      IF((D2-D1).EQ.0.0)D2 = D2 * 1.00001
      M = (Y2-Y1)/(D2-D1)
      B = Y1 - M*D1
      RES = (Y-B)/M
      RETURN
      END
*************************************************************
*        *
*     THIS SUBROUTINE EXTRAPOLATES THE ZSTAR(I) DATA         *
*     TO GENERATE ZSTAR(0) AND ZSTAR(S+1) FOR COMPUTATION    *
*     OF THE FIVE CRITICAL VALUES.                           *
*                                                            *
*************************************************************
      SUBROUTINE EXTRA(N,D)
      INTEGER N,NO,N1
      REAL Y1,Y2,D(0:10003),D1,D2,ZZ
      Y1 = 0.5/N
      Y2 = 1.5/N
      D1 = D(1)
      D2 = D(2)
      CALL CAN(Y1,Y2,D1,D2,0.0,ZZ)
      IF(ZZ.GE.0.0) THEN
        D(0) = ZZ
      ELSE
        D(0) = 0.0
      ENDIF
      Y1 = (REAL(N) - 1.5)/N
      Y2 = (REAL(N) - 0.5)/N
      NO = N-1
      D1 = D(NO)
      D2 = D(N)
      CALL CAN(Y1,Y2,D1,D2,1.0,ZZ)
      N1 = N + 1
      D(N1) = ZZ
      RETURN
      END
*************************************************************
*        *
*     THE FOLLOWING SUB DETERMINES THE %TILES AND FINDS      *
*     THE CRITICAL VALUES BY EVOKING THE SUB CAN             *
*        *
```

```
*****************************************************************
      SUBROUTINE VALUES(D,CRIT80,CRIT85,CRIT90,CRIT95,CRIT99,N)
      INTEGER I,N,NN
      REAL D(0:10003),Y(0:10003),C80,C90,C95,C99,C85,
     + Y79,D79,Y81,D81,DIF90,Y89,Y91,D89,D91,DIF95,DIF80,
     + Y94,Y96,D94,D96,DIF99,Y98,Y100,D98,D100,DIF85,
     + Y84,D84,Y86,D86,CRIT85,CRIT80,CRIT90,CRIT95,CRIT99
*
      DO 100 I = 1,N
        Y(I) = (REAL(I) - 0.5)/REAL(N)
 100    CONTINUE
      Y(0) = 0.0
         NN = N + 1
         Y(NN) = 1.0
      C80 = 1000.0
      C85 = 1000.0
      C90 = 1000.0
      C95 = 1000.0
      C99 = 1000.0
      DO 200 I = NN,0,-1
        IF (Y(I).LE.0.75) GO TO 300
        IF (Y(I).GT.0.75.AND.Y(I).LE.0.80) THEN

C          GET THE DESIRED %TILE AT 80%
          DIF80 = .80 - Y(I)
          IF (DIF80.LE.C80) THEN
            C80 = DIF80
            Y79 = Y(I)
            D79 = D(I)
            Y81 = Y(I+1)
            D81 = D(I+1)
          ENDIF
        ELSEIF (Y(I).GT.0.80.AND.Y(I).LE.0.85) THEN

C          GET THE DESIRED %TILE AT 85%
          DIF85 = .85 - Y(I)
          IF (DIF85.LE.C85) THEN
            C85 = DIF85
            Y84 = Y(I)
            D84 = D(I)
            Y86 = Y(I+1)
            D86 = D(I+1)
```

```
          ENDIF
        ELSEIF (Y(I).GT.0.85.AND.Y(I).LE.0.90) THEN

C         GET THE DESIRED %TILE AT 90%
          DIF90 = .90 - Y(I)
          IF (DIF90.LE.C90) THEN
            C90 = DIF90
            Y89 = Y(I)
            D89 = D(I)
            Y91 = Y(I+1)
            D91 = D(I+1)
          ENDIF
        ELSEIF (Y(I).GT.0.90.AND.Y(I).LE.0.95) THEN

C         GET THE DESIRED %TILE AT 95%
          DIF95 = .95 - Y(I)
          IF (DIF95.LE.C95) THEN
            C95 = DIF95
            Y94 = Y(I)
            D94 = D(I)
            Y96 = Y(I+1)
            D96 = D(I+1)
          ENDIF

        ELSEIF (Y(I).GT.0.95.AND.Y(I).LE.0.99) THEN

C         GET THE DESIRED %TILE AT 99%
          DIF99 = .99 - Y(I)
          IF (DIF99.LE.C99) THEN
            C99 = DIF99
            Y98 = Y(I)
            D98 = D(I)
            Y100 = Y(I+1)
            D100 = D(I+1)
          ENDIF
        ENDIF
200   CONTINUE

300   IF (DIF80.EQ.0.0) THEN
        CRIT80 = D79
      ELSE
C       COMPUTE THE CRIT VALUE AT SIGNIFICANCE LEVEL = .20
```

```
          CALL CAN(Y79,Y81,D79,D81,.80,CRIT80)
        ENDIF

        IF (DIF85.EQ.0.0) THEN
          CRIT85 = D84
        ELSE
C         COMPUTE THE CRIT VALUE AT SIGNIFICANCE LEVEL = .15
          CALL CAN(Y84,Y86,D84,D86,.85,CRIT85)
        ENDIF

        IF (DIF90.EQ.0.0) THEN
          CRIT90 = D89
        ELSE
C         COMPUTE THE CRIT VALUE AT SIGNIFICANCE LEVEL = .10
          CALL CAN(Y89,Y91,D89,D91,.90,CRIT90)
        ENDIF

        IF (DIF95.EQ.0.0) THEN
          CRIT95 = D94
        ELSE
C         COMPUTE THE CRIT VALUE AT SIGNIFICANCE LEVEL = .05
          CALL CAN(Y94,Y96,D94,D96,.95,CRIT95)
        ENDIF

        IF (DIF99.EQ.0.0) THEN
          CRIT99 = D98
        ELSE
C         COMPUTE THE CRIT VALUE AT SIGNIFICANCE LEVEL = .01
          CALL CAN(Y98,Y100,D98,D100,.99,CRIT99)
        ENDIF
        RETURN
        END
```

Appendix F. Fortran Program for Conducting the Power Study of the Z^* Test Statistic

```
*****************************************************************
  THIS PROGRAM COMPUTES THE POWER STUDY FOR THE Z star STATISTIC
*****************************************************************

       INTEGER I,J,IREPS,N,SEED,IERROR,NN
       INTEGER REJ(1:10,0:5),DIST,NUMDIST,II,JJ
*
       REAL Y(35),X(35),GAP(35),MUDIF(34)
       REAL XX(35),ZSTAR(1:5000),G(20)
       REAL NUMSUM,NUM,DENSUM,DENOM,K
       REAL CRIT(5)
       REAL PWR(10,5)
*
       DATA CRIT/[                                ]/
*
       EXTERNAL RNSET,SVRGN,RNWIB
       EXTERNAL RNUN,RNBET,RNNOR,RNGAM
*
*              set shape parameter for this run
       K= [k]
       IREPS=5000
       NUMDIST=10
*
OPEN(UNIT=[n],FILE='d[n]_[k].prn',STATUS='OLD',IOSTAT=
     +               IERROR, ERR=1000)
*
       OPEN(UNIT=33,FILE='mudif.out')
       OPEN(UNIT=44,FILE='yvals.dat')
       OPEN(UNIT=55,FILE='xvals.dat')
       OPEN(UNIT=66,FILE='xxvals.dat')
       OPEN(UNIT=77,FILE='gaps.dat')
       OPEN(UNIT=88,FILE='gi.dat')
       OPEN(UNIT=99,FILE='zstar.dat')
*
       OPEN(UNIT=[n+1],FILE='p[k]n[n].tex')
*
       SEED=60486438
       CALL RNSET(SEED)
```

```
*
      N= [n]
            I=0
            J=0
            NN=N+1
            DO 5 I=1,N
                X(I)=0.0
                Y(I)=0.0
                GAP(I)=0.0
                G(I)=0.0
 5          CONTINUE
            DO 71 II=1,10
              DO 71 I=0,5
                REJ(II,I)=0
 71         CONTINUE
            DO 81 JJ=1,10
              DO 81 J=1,5
                PWR(JJ,J)=0.0
 81         CONTINUE
*
      WRITE(NN,970)
970   FORMAT('\\documentstyle[12pt]{thesis}'/'\\begin{document}',/
     +'\\begin{table}[h]'/'\\caption{Power Study: Sample size nn,',
     + 'shape parameter kkk, alpha levels = 0.20,...0.01}',/
     + '\\begin{center}')
*
      READ(N,*)(MUDIF(I),I=1,N-1)
      WRITE(33,903)(MUDIF(I),I=1,N-1)
903   FORMAT('EXP.VAL.GAPS ',/,10F8.5,/,9F8.5/)
*
      WRITE(NN,971)
971   FORMAT('\\begin{tabular}{|c|c|c|c|c|c|} \\hline',/,
     +6X,' & 0.20 & 0.15 & 0.10 & 0.05 & 0.01 \\','\\ \\hline',
     +'\\ \\hline',/,' DIST ',5('&',6X),'\\','\\ \\hline')
*
         DO 100 DIST=1,NUMDIST
*
              DO 6 J=1,IREPS
                  ZSTAR(J)=0.0
 6            CONTINUE
*
              DO 110 J=1,IREPS
```

```
*
              GO TO(200,210,220,230,240,250,
     +              260,270,280,290)DIST
*
          WRITE(*,101)
 101      FORMAT( 'ERROR AT COMPUTED GOTO')
          STOP
*
 200      CALL RNWIB(N,K,Y)
          GO TO 440
*
 210      CALL RNWIB(N,2.0,Y)
          GO TO 440
*
 220      CALL RNWIB(N,3.5,Y)
          GO TO 440
*
 230      CALL RNGAM(N,1.0,Y)
          GO TO 440
*
 240      CALL RNGAM(N,2.0,Y)
          GO TO 440
*
 250      CALL RNUN(N,Y)
          GO TO 440
*
 260      CALL RNNOR(N,Y)
          GO TO 440
*
 270      CALL RNBET(N,2.0,3.0,Y)
          GO TO 440
*
 280      CALL RNBET(N,2.0,2.0,Y)
          GO TO 440
*
 290      CALL RNBET(N,1.0,1.0,Y)
*
 440      WRITE(44,904)(Y(I),I=1,N)
 904      FORMAT(1X,' RAW DATA SET IS',/,
     +                    5(4F10.6)/)
*
          DO 7 I=1,N
```

```
            X(I)=Y(I)+10.0
7       CONTINUE
*
        WRITE(55,905)(X(I),I=1,N)
905     FORMAT(1X,' SCALED DATA SET IS',/,
     +                5(4F10.6)/)
*
        CALL SVRGN(N,X,XX)
        WRITE(66,906)(XX(I),I=1,N)
906     FORMAT(1X,' SORTED DATA SET IS',/,
     +                5(4F10.6)/)
*
        DO 8 I=1,N-1
            GAP(I)=XX(I+1)-XX(I)
8       CONTINUE
        WRITE(77,907)(GAP(I),I=1,N-1)
907     FORMAT(1X,'GAPS ARE ',/,5(4F10.6),/)
*
        DO 9 I=1,N-1
            G(I)=GAP(I)/MUDIF(I)
9       CONTINUE
        WRITE(88,908)(G(I),I=1,N-1)
908     FORMAT(1X,'G(I) VALS ',/,5(4F10.6),/)
*
        NUMSUM=0.0
*
        DO 10 I=1,N-2
        NUMSUM=NUMSUM+(N-1-I)*G(I)
10      CONTINUE
        NUM=2.0*NUMSUM
*
        DENSUM=0.0
*
        DO 11 I=1,N-1
        DENSUM=DENSUM+G(I)
11      CONTINUE
        DENOM=(N-2)*DENSUM
*
        ZSTAR(J)=NUM/DENOM
*
        WRITE(99,909) ZSTAR(J)
909     FORMAT(1X,'Z* VALUE IS ' ,F10.6)
```

```
*
 110          CONTINUE
*
          CALL SVRGN(IREPS,ZSTAR,ZSTAR)
*
          DO 120 I=1,IREPS
*
              IF(ZSTAR(I).GT.CRIT(5)) THEN
                  DO 99 J=1,5
                       REJ(DIST,J)=REJ(DIST,J)+1
  99              CONTINUE
                  ELSEIF(ZSTAR(I).GT.CRIT(4)) THEN
                  DO 95 J=1,4
                       REJ(DIST,J)=REJ(DIST,J)+1
  95              CONTINUE
              ELSEIF(ZSTAR(I).GT.CRIT(3)) THEN
                       DO 90 J=1,3
                       REJ(DIST,J)=REJ(DIST,J)+1
  90              CONTINUE
              ELSEIF(ZSTAR(I).GT.CRIT(2)) THEN
                  DO 85 J=1,2
                       REJ(DIST,J)=REJ(DIST,J)+1
  85              CONTINUE
              ELSEIF(ZSTAR(I).GT.CRIT(1)) THEN
                       REJ(DIST,1)=REJ(DIST,1)+1
              ELSE
                       REJ(DIST,0)=REJ(DIST,0)+1
              ENDIF
*
 120      CONTINUE
*
C             WRITE(NN,971)DIST,(REJ(DIST,I),I=0,5)
C         971     FORMAT(1X,'REJ TOTALS FOR DIST =  ',I2,3X,
C           +                 ' ARE  ',/,6I8/)
*
              DO 130 J=1,5
                    PWR(DIST,J)=(REAL(REJ(DIST,J))/
    +                               REAL(IREPS))
 130          CONTINUE
*
         WRITE(NN,972)DIST,(PWR(DIST,J),J=1,5)
 972     FORMAT(I2,5(' & ',F6.3),'\\','\\ \\hline')
```

```
*
C 972      FORMAT(1X,' POWER FOR DIST =  ',I2,3X,
C     +                 ' ARE  ',/,5X,5F9.3/)
*
 100       CONTINUE
*
       CLOSE(N)
*
       WRITE(NN,973)
 973   FORMAT('\\end{tabular}'/'\\end{center}'/'\\end{table}'/,
     + '\\end{document}')
*
       CLOSE(NN)
*
      STOP
*
 1000  WRITE(*,1001) IERROR
 1001  FORMAT(1X,'IOSTAT = ', I3/)
*
      END
```

Vita

Mark Charles Coppa was born on 1 September 1959 in New York City. He graduated from Cardinal Spellman High School in Bronx, New York in 1977. He attended the Stevens Institute of Technology in Hoboken, New Jersey and received the bachelor's degree in Applied Mathematics in 1981. After commissioning through the AFROTC program, he attended Undergraduate Navigator Training at Mather AFB, California. After receiving his wings in 1982, his first flying assignment was in the C-5 Galaxy at Dover AFB, Delaware. Subsequent flying assignments were in the WC-135B at McClellan AFB, California and in the VC-135 at Ramstein AB, Germany. He entered the School of Engineering, Air Force Institute of Technology in August of 1991.

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	March 1993	Master's Thesis

4. TITLE AND SUBTITLE
A NEW GOODNESS-OF-FIT TEST FOR THE WEIBULL DISTRIBUTION BASED ON SPACINGS

5. FUNDING NUMBERS

6. AUTHOR(S)
Mark Charles Coppa

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Air Force Institute of Technology
WPAFB OH 45433-6583

8. PERFORMING ORGANIZATION REPORT NUMBER
AFIT/GST/ENS/93M-02

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Distribution Unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*
The critical values for a new goodness-of-fit test based on spacings are generated for the Weibull distribution when the shape parameter is known. The critical values are used for testing whether a set of observations follow a Weibull distribution when the scale and location parameters are unknown. A Monte Carlo simulation with 10,000 iterations is used to generate the critical values for sample sizes 5(5)35 at shape parameters k equal to 0.5(0.5)1.5 and for sample sizes 5(5)20 at shape parameters k = 2.0(1.0)4.0. A Monte Carlo power study of the Z^* test statistic using 5000 iterations is accomplished using nine alternate distributions H_a. The power is good to excellent when the null hypothesis H_0 is from a skewed distribution($k < 2.0$). Power results at shape parameters $k \geq 2.0$ are poor for all sample sizes considered. A comparison is made, at shape parameter 1.0, against the prominent competing goodness-of-fit test statistics. Data is obtained from a prior AFIT thesis by Bush. Results indicate that the Z^* test is more powerful than the competition at the available sample sizes of 5, 15 and 25 and α levels: 0.05 and 0.01. A relationship between the critical value and the sample size is investigated to allow for greater usage of the test statistic. Satisfactory values of *fit* are attained with a simple log-linear relationship.

14. SUBJECT TERMS
Goodness-of-Fit Test, Weibull Distribution, Spacings

15. NUMBER OF PAGES
114

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL